

PHARAON, [s.m]. (Game of chance.) the main rules of this game are,

That the banker cuts with a whole deck composed of fifty-two cards.

That he draws all the cards in succession, putting the ones to his right, & the others to his left.

That to each hand one cuts, that is to say of two by two cards: the punter has the liberty of taking one or several cards, & of risking on them a certain sum.

That the banker earns the stake of the punter, when the card of the punter arrives to the right hand on an odd place, & that he loses, when the card of the punter falls to the left hand, & in an even place.

That the banker takes the half of this that the punter has bet on the card, when in a same cut, the card of the punter occurs two times; this which makes a part of the advantage of the banker.

And finally that the last card which should be for the punter, is neither for him, nor for the banker; this which is again an advantage for the banker;

Whence one sees, 1°. that the card of the punter is not being more than one times in the talon, the difference of the sort of the banker & of the punter is founded on that which between all the various possible arrangements of the cards of the banker, there is a greater number of them which make to earn it, than there are of them which make to lose it, the last card being considered as null; 2°. that the advantage of the banker increases in measure as the number of cards of the banker decreases; 3°. that the card of the punter being two times in the talon, the advantage of the banker draws from the probability that there is that the card of the punter will come twice in a same tally; because then the banker wins half of the stake of the punter, excepting the only case where the card of the punter would come in doublet in the last cut, this which would give to the banker the entire wager of the punter; 4°. that the card of the punter being three or four times in the hand of banker, [p. 12:487] the advantage of banker is found on the possibility that there is that the card of the punter is found twice in a same tally, before it has come as pure gain or as pure loss for the banker. Now this possibility increases or decreases, according as there are more or less cards in the hand of the banker, & according as the map of the punter is found there more or less times.

Whence one concludes again that for knowing the advantage of the banker, by ratio to the punters, in all the different circumstances of the game, it is necessary to discover in all the different possible arrangements of the cards that the banker holds, & in the supposition that the card is found there either one, or two, or three, or four times, which are those which makes a win, and which are those which give to him the half of the stake of the punter, which are those which make a loss, & which are those finally which make neither a loss nor a win.

One is able to form two tables¹ of all these different chances. For in knowing the usage, in the first, the number contained in the cell would express the number of cards that the banker holds, & the number which follows, either the cell in the first column, or two points in the other columns, would express the number of times that the card of the punter is supposed to be found in the hand of banker.

The usage of the second table would be to give some expressions, to the truth less exact, but simpler & more intelligible to the players: in order to understand this table, it is necessary to know that this sign > mark excess, & that this one < marks defect; in a way that $> 1/4 < 1/3$ means greater than $1/4$, & smaller than $1/3$.

In examining these tables, one would see in the first column that the advantage to the banker is expressed in the first column by a fraction of which the numerator being always the unit, the denominator is the number of the cards that the banker holds.

In the second column, that this advantage is expressed by a fraction of which the numerator being according to the sequence of natural numbers, 1, 2, 3, 4, &c. the denominator has for difference between these terms, the numbers 8, 26, 34, 42, 50, 58, which the difference is 8.

That in the third column the numerator always being 3, the difference which rules in the denominator is 8.

That in the fourth column the difference always being 4 in the numerator, the denominator has for difference between its terms the numbers 24, 40, 56, 72, 88, & of which the difference is 16.

That another uniformity singular enough between the last numbers of the denominator of each term of one column, it is that in the first the last numbers of denominator are according to this order: 4, 6, 8, 0, 2, | 4, 6, 8, 0, 2; & in the second according to this order, 2, 0, 6, 0, 2, | 2, 0, 6, 0, 2, | 2, 0, 6, 0, 2; & in the third

¹ These tables are taken directly from Montmort's *Essai*.

according to this order, 2, 0, 8, 6, 4, | 2, 0, 8, 6, 4: & in the fourth according to this order, 6, 0, 0, 6, 8, | 6, 0, 0, 6, 8, &c.

One may, by the means of these tables, find all one trial how much a banker has of advantage on each card, how much each complete cut will have due, to equal fortune, to bring profit to the banker, if one remembers the number of cards taken by the punters, of the various circumstances in which one has wagered on the game by them, & finally of the quantity of money risked on these cards.

One would give of fair limits to this advantage, in establishing that the doublets are indifferent for the banker & for the punter, or at least that they are worth only to the banker the third or the quarter of the stake of the punter.

So that the punter taking a card has the least possible disadvantage, it is necessary that he chooses one which has passed twice; there would be more disadvantage for him, if he would take a card which has passed one time; more again on a card which has been passed three times, & the worst choice would be a card which would not pass again.

So, in supposing $A =$ one pistole, the advantage of the banker who would be 19 sols 2 deniers, under the supposition that the card of punter would be four times in twelve cards, will become 16 sols 8 deniers if it is there only one time; 13 sols 7 deniers if it is there three times; & 10 sols 7 deniers if it is there only two times.

The persons who have not examined the substance of the game will ask why one has said nothing of the masses, on the parolis, of the paix, & on the sept & it goes, it is that all this signifies nothing, that one risks more or less, & then this is all; the odds do not change.

The advantage of banker increases in proportion as the number of his cards decreases.

The advantage of banker on a card which has not passed, is nearly double of the one that he has on a card which has passed twice; his advantage on a card which has passed three times is to his advantage on a card which has passed two times in a greater ratio than of three to two.

The advantage of the banker who would be only about 24 sols if the punter wagered six pistoles or to the first cut of game, or on a card which would have passed twice, when there would remain not more than twenty-eight in the hand of banker (for these two cases returns near enough to the same thing) will be 7 livres 2 sols if the punter wagers six pistoles on a card which has not passed again, the talon being composed only of ten cards.

The advantage of banker would be precisely six livres, if the card of the punter, in this last case, passes three times.

So, all the knowledge of *pharaon* reduces for the punters to the observation of the two following rules.

Do not take cards that are in the first cut & wager on the game accordingly unless there are a greater number of passed cuts.

To regard as the worst cards those which have not passed again, or which have passed three times, & to prefer to all those which have passed two times.

It is thus that the punter will render his disadvantage the least possible.

52	$1 = * * *$	$: 2 = * * *$	$: 3 = * * *$	$: 4 = a + \frac{2295086253}{115890841950} a$
50	$1 = * * *$	$: 2 = a + \frac{3117}{350350} a$	$: 3 = a + \frac{3}{196} a$	$: 4 = a + \frac{8208829}{349595300} a$
48	$1 = a + \frac{1}{48} a$	$: 2 = a + \frac{1787}{161304} a$	$: 3 = a + \frac{3}{188} a$	$: 4 = a + \frac{276199}{12842280} a$
46	$1 = a + \frac{1}{46} a$	$: 2 = a + \frac{3431}{296010} a$	$: 3 = a + \frac{3}{180} a$	$: 4 = a + \frac{11002}{489555} a$
44	$1 = a + \frac{1}{44} a$	$: 2 = a + \frac{822}{67639} a$	$: 3 = a + \frac{3}{172} a$	$: 4 = a + \frac{913}{38786} a$
42	$1 = a + \frac{1}{42} a$	$: 2 = a + \frac{3145}{246246} a$	$: 3 = a + \frac{3}{164} a$	$: 4 = a + \frac{79}{3198} a$
40	$1 = a + \frac{1}{40} a$	$: 2 = a + \frac{1501}{111540} a$	$: 3 = a + \frac{3}{156} a$	$: 4 = a + \frac{25}{962} a$
38	$1 = a + \frac{1}{38} a$	$: 2 = a + \frac{2849}{201068} a$	$: 3 = a + \frac{3}{148} a$	$: 4 = a + \frac{1349}{49210} a$
36	$1 = a + \frac{1}{36} a$	$: 2 = a + \frac{679}{45045} a$	$: 3 = a + \frac{3}{140} a$	$: 4 = a + \frac{1139}{39270} a$
34	$1 = a + \frac{1}{34} a$	$: 2 = a + \frac{2573}{160446} a$	$: 3 = a + \frac{3}{132} a$	$: 4 = a + \frac{357}{11594} a$
32	$1 = a + \frac{1}{32} a$	$: 2 = a + \frac{1215}{70928} a$	$: 3 = a + \frac{3}{124} a$	$: 4 = a + \frac{177}{5394} a$
30	$1 = a + \frac{1}{30} a$	$: 2 = a + \frac{2287}{124410} a$	$: 3 = a + \frac{3}{116} a$	$: 4 = a + \frac{55}{1566} a$
28	$1 = a + \frac{1}{28} a$	$: 2 = a + \frac{536}{27027} a$	$: 3 = a + \frac{3}{108} a$	$: 4 = a + \frac{221}{5850} a$
26	$1 = a + \frac{1}{26} a$	$: 2 = a + \frac{2001}{92950} a$	$: 3 = a + \frac{3}{100} a$	$: 4 = a + \frac{611}{14950} a$
24	$1 = a + \frac{1}{24} a$	$: 2 = a + \frac{929}{39468} a$	$: 3 = a + \frac{3}{92} a$	$: 4 = a + \frac{473}{10626} a$
22	$1 = a + \frac{1}{22} a$	$: 2 = a + \frac{1715}{66066} a$	$: 3 = a + \frac{3}{84} a$	$: 4 = a + \frac{143}{2926} a$
20	$1 = a + \frac{1}{20} a$	$: 2 = a + \frac{1572}{54340} a$	$: 3 = a + \frac{3}{76} a$	$: 4 = a + \frac{35}{646} a$
18	$1 = a + \frac{1}{18} a$	$: 2 = a + \frac{1429}{43758} a$	$: 3 = a + \frac{3}{68} a$	$: 4 = a + \frac{31}{510} a$
16	$1 = a + \frac{1}{16} a$	$: 2 = a + \frac{429}{11440} a$	$: 3 = a + \frac{3}{60} a$	$: 4 = a + \frac{9}{130} a$
14	$1 = a + \frac{1}{14} a$	$: 2 = a + \frac{44}{1001} a$	$: 3 = a + \frac{3}{52} a$	$: 4 = a + \frac{23}{286} a$
12	$1 = a + \frac{1}{12} a$	$: 2 = a + \frac{7}{132} a$	$: 3 = a + \frac{3}{44} a$	$: 4 = a + \frac{19}{198} a$
10	$1 = a + \frac{1}{10} a$	$: 2 = a + \frac{1}{15} a$	$: 3 = a + \frac{3}{36} a$	$: 4 = a + \frac{5}{42} a$
8	$1 = a + \frac{1}{8} a$	$: 2 = a + \frac{5}{56} a$	$: 3 = a + \frac{3}{28} a$	$: 4 = a + \frac{11}{70} a$
6	$1 = a + \frac{1}{6} a$	$: 2 = a + \frac{2}{15} a$	$: 3 = a + \frac{3}{20} a$	$: 4 = a + \frac{7}{30} a$
4	$1 = a + \frac{1}{4} a$	$: 2 = a + \frac{1}{4} a$	$: 3 = a + \frac{3}{12} a$	$: 4 = a + \frac{1}{2} a$

TABLE II. FOR PHARAON.

52	$1 = * * *$	$: 2 = * * *$	$: 3 = * * *$	$: 4 = a + > \frac{1}{51} < \frac{1}{50}$
50	$1 = * * *$	$: 2 = a + > \frac{1}{95} < \frac{1}{94}$	$: 3 = a + > \frac{1}{66} < \frac{1}{65}$	$: 4 = a + > \frac{1}{49} < \frac{1}{48}$
48	$1 = a + \frac{1}{48}a$	$: 2 = a + > \frac{1}{91} < \frac{1}{90}$	$: 3 = a + > \frac{1}{63} < \frac{1}{62}$	$: 4 = a + > \frac{1}{47} < \frac{1}{46}$
46	$1 = a + \frac{1}{46}a$	$: 2 = a + > \frac{1}{87} < \frac{1}{86}$	$: 3 = a + \frac{1}{60}$	$: 4 = a + > \frac{1}{45} < \frac{1}{44}$
44	$1 = a + \frac{1}{44}a$	$: 2 = a + > \frac{1}{83} < \frac{1}{82}$	$: 3 = a + > \frac{1}{58} < \frac{1}{57}$	$: 4 = a + > \frac{1}{43} < \frac{1}{42}$
42	$1 = a + \frac{1}{42}a$	$: 2 = a + > \frac{1}{79} < \frac{1}{78}$	$: 3 = a + > \frac{1}{55} < \frac{1}{54}$	$: 4 = a + > \frac{1}{41} < \frac{1}{40}$
40	$1 = a + \frac{1}{40}a$	$: 2 = a + > \frac{1}{75} < \frac{1}{74}$	$: 3 = a + \frac{1}{52}$	$: 4 = a + > \frac{1}{39} < \frac{1}{38}$
38	$1 = a + \frac{1}{38}a$	$: 2 = a + > \frac{1}{71} < \frac{1}{70}$	$: 3 = a + > \frac{1}{50} < \frac{1}{49}$	$: 4 = a + > \frac{1}{37} < \frac{1}{36}$
36	$1 = a + \frac{1}{36}a$	$: 2 = a + > \frac{1}{67} < \frac{1}{66}$	$: 3 = a + > \frac{1}{47} < \frac{1}{46}$	$: 4 = a + > \frac{1}{35} < \frac{1}{34}$
34	$1 = a + \frac{1}{34}a$	$: 2 = a + > \frac{1}{63} < \frac{1}{62}$	$: 3 = a + \frac{1}{44}$	$: 4 = a + > \frac{1}{33} < \frac{1}{32}$
32	$1 = a + \frac{1}{32}a$	$: 2 = a + > \frac{1}{59} < \frac{1}{58}$	$: 3 = a + > \frac{1}{42} < \frac{1}{41}$	$: 4 = a + > \frac{1}{31} < \frac{1}{30}$
30	$1 = a + \frac{1}{30}a$	$: 2 = a + > \frac{1}{55} < \frac{1}{54}$	$: 3 = a + > \frac{1}{39} < \frac{1}{38}$	$: 4 = a + > \frac{1}{29} < \frac{1}{28}$
28	$1 = a + \frac{1}{28}a$	$: 2 = a + > \frac{1}{51} < \frac{1}{50}$	$: 3 = a + \frac{1}{36}$	$: 4 = a + > \frac{1}{27} < \frac{1}{26}$
26	$1 = a + \frac{1}{26}a$	$: 2 = a + > \frac{1}{47} < \frac{1}{46}$	$: 3 = a + > \frac{1}{34} < \frac{1}{33}$	$: 4 = a + > \frac{1}{25} < \frac{1}{24}$
24	$1 = a + \frac{1}{24}a$	$: 2 = a + > \frac{1}{43} < \frac{1}{42}$	$: 3 = a + > \frac{1}{31} < \frac{1}{30}$	$: 4 = a + > \frac{1}{23} < \frac{1}{22}$
22	$1 = a + \frac{1}{22}a$	$: 2 = a + > \frac{1}{39} < \frac{1}{38}$	$: 3 = a + \frac{1}{28}$	$: 4 = a + > \frac{1}{21} < \frac{1}{20}$
20	$1 = a + \frac{1}{20}a$	$: 2 = a + > \frac{1}{35} < \frac{1}{34}$	$: 3 = a + > \frac{1}{26} < \frac{1}{25}$	$: 4 = a + > \frac{1}{19} < \frac{1}{18}$
18	$1 = a + \frac{1}{18}a$	$: 2 = a + > \frac{1}{31} < \frac{1}{30}$	$: 3 = a + > \frac{1}{23} < \frac{1}{22}$	$: 4 = a + > \frac{1}{17} < \frac{1}{16}$
16	$1 = a + \frac{1}{16}a$	$: 2 = a + > \frac{1}{27} < \frac{1}{26}$	$: 3 = a + \frac{1}{20}$	$: 4 = a + > \frac{1}{15} < \frac{1}{14}$
14	$1 = a + \frac{1}{14}a$	$: 2 = a + > \frac{1}{23} < \frac{1}{22}$	$: 3 = a + > \frac{1}{18} < \frac{1}{17}$	$: 4 = a + > \frac{1}{13} < \frac{1}{12}$
12	$1 = a + \frac{1}{12}a$	$: 2 = a + > \frac{1}{19} < \frac{1}{18}$	$: 3 = a + > \frac{1}{15} < \frac{1}{14}$	$: 4 = a + > \frac{1}{11} < \frac{1}{10}$
10	$1 = a + \frac{1}{10}a$	$: 2 = a + \frac{1}{15}$	$: 3 = a + \frac{1}{12}$	$: 4 = a + > \frac{1}{9} < \frac{1}{8}$
8	$1 = a + \frac{1}{8}a$	$: 2 = a + > \frac{1}{12} < \frac{1}{11}$	$: 3 = a + > \frac{1}{10} < \frac{1}{9}$	$: 4 = a + > \frac{1}{7} < \frac{1}{6}$
6	$1 = a + \frac{1}{6}a$	$: 2 = a + > \frac{1}{8} < \frac{1}{7}$	$: 3 = a + > \frac{1}{7} < \frac{1}{6}$	$: 4 = a + > \frac{1}{5} < \frac{1}{4}$
4	$1 = a + \frac{1}{4}a$	$: 2 = a + \frac{1}{4}$	$: 3 = a + \frac{1}{4}a$	$: 4 = a + \frac{1}{2}a$