

PROBABILITÉ

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PROBABILITY, *Philosophy, Logic, Mathematics.* Every proposition considered in itself is true or false; but relative to us, it can be certain; we can perceive more or less the relations which can exist between two ideas, or the suitability of the one with the other, based on certain conditions which link them, & which, when they are all known to us, give us the certitude of that truth, or of that proposition; but if we know only a part of it, we have then only a simple *probability*, which has the more truth, as we are assured of a greater number of these conditions. These are they which form the degrees of *probability*, of which a correct estimate & an exact measure could make the completion of sagacity & of prudence.

Geometers have judged that their calculus can serve to evaluate these degrees of *probability*, at least to a certain point, & they had recourse to logic, or to the art of reasoning, in order to discover the principles of it & to establish the theory of it. They have regarded certitude as this whole, & the *probabilities* as the parts of this whole. Consequently the correct degree of *probability* of a proposition has been exactly known to them, when they have been able to say & prove that this *probability* is worth a half, a fourth, or a third of certitude. Often they are content to assume it; their calculus in itself is not less correct, & these expressions, which can appear a little bizarre, are not less meaningful. Some examples taken from games, from wagers or from assurances, will clarify them. We suppose that one just said to me that I have had in a lottery a lot of ten thousand livres, I doubt the truth of this news. Someone, who is present, asks me what sum I would wish to give in order that he insured it to me. I offer him the half, that which suggests that I regard the probability of this news, only as a half-certitude; but if I had offered only one thousand livres, it had been to say that I have nine times more reason to believe the truth than to not believe it. Now this

would make the probability carry to nine degrees, in a manner that certitude having ten, it would miss only one in order to add one whole proof to the news.

In ordinary usage, we call *probable*¹ that which has more than half-certitude, *credible*², that which surpasses it considerably, & *morally certain*³, that which touches whole certitude. We speak here only of moral certitude, which nearly coincides with mathematical certitude, although it is not susceptible to the same proofs. Moral evidence is therefore properly only a probability so great, that it is of a wise man to think and to act, in the case where one has this certitude, as one must think & act, if one had a mathematical one. It is of a moral certitude that there is a city of Rome; the contrary implies no contradiction; it is not impossible that all those who say to me to have seen it, do not agree to be mistaken, that the books which speak of it are not done on purpose for this, that the monuments which we have of it, are not supposed; however, if I would refuse to render to me to an evidence supported on the proofs that I have of the evidence of Rome, simply because they are not susceptibles of mathematical demonstration, one would treat me, with reason, of insanity, because the *probability* that there is a city of Rome, so strongly surpasses over the suspicion that it can be there not at all, that at pain would one express by number this difference or the value of this *probability*. This example suffices to demonstrate the moral evidence & its degrees which are so much of *probabilities*. A half-certitude forms the *uncertain* properly said, where the mind finding on all sides equal reasons, does not make what judgment to carry, what part to take. In this state of equilibrium, the slightest proof to us determines; frequently we seek it where there is neither reason nor wisdom to seek it; & as it is rather difficult in many cases, where the opposite reasons approach nearly to equality, to determine what are those which must surpass it, the wisest men extend the point of uncertainty; they do not fix it only at this state of the soul, where it is equally driven on all sides by the weight of the reasons; but they carry it yet onto each situation which approaches it rather in order that we can not perceive inequality; it happens thence that the land of uncertainty is more or less vast, according to the degree more or less of knowledge, of logic & of courage. It is more compact to those who are the wisest or least wise; because temerity confines it more than prudence, by the hardest of their decisions. Under this half-certitude or uncertainty, they find *suspicion* and *doubt*, which terminate at the certitude of falsity of a proposition. A thing is false of a moral evidence, when the probability of its existence is so

¹ The French word is *probable*.

² The French word is *vraisemblable*.

³ The French phrase is *moralement certain*.

much inferior to the contrary *probability*, that it has ten thousand, one hundred thousand odds against one that it is not.

Here are the degrees of *probability* between the two opposed evidences. Before researching the sources of them, it will not be useless in an article where we do not wish to content ourselves of simple geometric calculus, to establish some general rules, which are regularly observed by wise & prudent persons.

1. It is contrary to reason to seek probabilities, & to content ourselves of them there where we can attain evidence. One would scoff at a mathematician, who, in order to prove a proposition of geometry, would have recourse to some opinions, to some credibles, when he could bring its demonstration; or at a judge who would prefer to divine by the past life of a criminal, if he is culpable, rather than to hear his confession, by which he avows his crime.
2. It does not suffice to examine one or two of the proofs which one can put forward of it, it is necessary to weigh in the balance to examine all those which can come to our knowledge, & to serve to discover the truth. If we demand what probability there is that a man aged 50 years dies in the year, it does not suffice to consider that in general of one hundred persons of 50 years, there die of them around 3 or 4 in a year, & to conclude that there are odds 96 against 4, or 24 against one; it is necessary yet to pay attention to the temperament of this man there, to the actual state of his sanity, to his kind of life, to his profession, to the country which he inhabits; each so many circumstances which influence on the duration of his life.
3. There are not enough proofs which serve to establish a truth, it is still necessary to examine those which combat it. One demands if a woman known & absent from her country since 25 years, of whom one has had no news, must be regarded as dead? On one side one says that, in spite of all sorts of researches one has learned nothing of her; that as voyager she could have been exposed to a thousand dangers, that a malady could have removed her in a place where she was unknown; that if she were alive, she would not neglect to give news of her, especially before presuming that she would have an inheritance to collect, & other reasons that we can allege. But, in these considerations, one opposes some others which must not be neglected. One says that the one of whom there is question is an indolent woman, who, on other occasions has not at all written, that perhaps her letters are lost, that it can be impossible to write. This which suffices to

show that in all things it is necessary to weigh the proofs, the *probabilities* on all sides, to oppose the ones to the others, because a very probable proposition can be false, & that makes of it *probability*, there is none of them at all so strong that it can not be fought & destroyed by a contrary yet stronger. Thence the opposition that one sees every day among the judgments of men. Thence most of the disputes which would end soon, if one wanted to not regard as evident that which is only probable, to listen & to weigh the reasons that one opposes our opinion.

4. Is it necessary to caution that in our judgments it is prudent to give consent to any proposition only in proportion to its degree of credibility? Who would observe this general rule, would have all the rightness of mind, all the prudence, all the wisdom possible. But that we are quite alienated from it! The most common minds can, with some attention, discern the true from the false; others who have more penetration, know how to distinguish the probable from the uncertain or from the doubtful; but there are only the geniuses distinguished by their wisdom who can assign to each proposition its correct degree of credibility, & to proportion their opinion: ah but these geniuses are rare!
5. Moreover, the wise & prudent man will not consider only the *probability* of success, he will weigh still the grandeur of good or of ill that one must attend by taking such part of it, or determining for the contrary of it, or by remaining in inaction; he will prefer even the one where he knows that the appearance of success is quite slight, when he sees at the same time that the risk that he incurs is nothing or a quite small thing; & that to the contrary, if he succeeds, he can obtain a quite considerable wealth.
6. Since it is not possible to fix, with that precision what would be desired, the degrees of *probability*, we content ourselves of very nearly what we can obtain. Sometimes, by a delicateness badly understood, one is exposes oneself, & society, to some worse evils than those that one would wish to avoid; it is an art to know how to be alienated from perfection in certain articles, in order to approach disadvantage in some more essential & more interesting others.
7. Finally it seems useless to add here that in uncertainty we must suspend decision & acting until we have more knowledge, but that if the case is such that it permits no delay, it is necessary to stop at that which will

appear most probable; & a time the decision that we have judged the most wise being taken, it is not necessary to repent it, even though the event would correspond in nothing to that which we had good reason to expect. If, in a fire, we can escape only by jumping through a window, it is necessary to determine for this decision, however injurious that it is. The uncertainty would be worse still, & whatever be the issue of it, we have taken the wisest decision, it is not at all necessary to have regret.

After these general rules of which it will be easy to make application, we come to the sources of *probability*. We reduce them to two kinds: the one contains the *probabilities* drawn from the consideration of the same nature, & of the number of causes or of the reasons which can influence on the truth of the proposition of which there is question: the other is founded only on past experience which can make us draw with confidence some conjectures for the future, when at least that we are assured that the same causes which have produced the past still exist, & are ascribed to produce the future.

An example will make better understood the nature and the difference of these two sources of *probability*. I suppose that one knows that one has placed into an urn thirty thousand tickets, among which there are ten thousand blacks & twenty thousand whites, & that one demands what is the *probability* that by drawing one from it at random, it will bring forth white? I say that by the sole consideration of the nature of things, & by comparing the number of causes which can make a white ticket come forth with the number of those which can make a black come forth, by that alone it is two times more probable that it will bring forth a white ticket than a black: such that as the ticket which just came forth, is necessarily either black or white, if one dividing this certitude into three degrees or equal parts, one will say that there are two degrees of *probability* to draw a white ticket, & one degree for the black ticket, or that the *probability* of a white ticket is $\frac{2}{3}$ of certitude, & that of the black ticket $\frac{1}{3}$ of this certitude.

But, we suppose that I see in the urn only a great number of tickets, without knowing the proportion there is of the white to the blacks, or even without knowing if there is none at all of a third color, in this case, how to determine the probability to draw a white from it? I say that this will be by making some tests, that is to say, by drawing a ticket in order to see what it will be, then returning it into the urn, by drawing a second which I return also, then a third, a fourth, & so forth as many as I would wish. It is clear that the first ticket drawn being come up white, gives only a very slight *probability* that the number of whites surpasses that of the blacks, a second white drawn would increase this *probability*, a third

would fortify it. Finally, if I drew from it in sequence a great number of whites, I will be right to conclude that they are all whites, & that with so much more credibility as I would have more tickets drawn. But, if on the first three tickets, I draw from it two whites & one black, I then say that there is some probability quite slight, that there are two times more whites than blacks. If, out of ten tickets, there are drawn four whites & six blacks, the *probability* increases, & it will increase in measure as the number of tests or of experiences will confirm to me always the same proportion of whites to the blacks. If I would have three thousand tests, & if I had two thousand white tickets against one thousand black, it would be nearly that the *probability* of drawing a white is the double of that of drawing a black.

This manner to determine probably the ratio of the causes which give birth to an event to those which make it fail, or more generally the proportion of the reasons or conditions which establish the truth of a proposition with those which give the contrary, is applied to all that which can happen or not happen, to all that which can be or not be. When I see on the death registers, that during twenty, fifty years, of the number of infants who are born, there die the third of them before the age of six years, I will conclude of a newly born infant, that the probability that he will attain at least the age of six years, is the $\frac{2}{3}$ of certitude. If I see that of two players who play with equal marbles, the first wins always two games, while the other wins only one, I will conclude with much *probability* that he is two times stronger than his antagonist; if I notice that some one of one hundred times that he has spoken to me, he has lied to me on ten occasions, the *probability* of his testimony will be in my mind only the $\frac{9}{10}$ of certitude or even less.

The attention given in the past, the fidelity of the memory to retain that which has arrived & the exactitude of the registers to preserve the events, make that which we call in the world *experience*. A man who has *experience* is the one who having seen much & reflected much, can say to you very nearly (because here there is no question of a mathematical precision) what *probability* there is that such event having arrived, such other will follow it; thus all things equal besides, the more one has made of proofs or experiences, & the more one is assured of the precise ratio of the number of the favorable causes to the number of contrary causes.

One could demand if this *probability* increasing to infinity by a sequence of repeated experiments, can become in the end a moral certainty; or if these increases are so limited, that diminishing gradually, they make at infinity only a finite *probability*. Because we know that they have some increases which,

although perpetual, make nonetheless at infinity only a finite sum; for example, if the first experience would give a *probability* which is only $\frac{1}{3}$ of certitude, & the second a *probability* which is only the third of this third, & the third a *probability* which is only the third of the second, & the fourth a *probability* which is only the third of the third, & thus to infinity. It would be easy, by the calculus, to see that all these *probabilities* together are only a half-certitude, so that one would have made in vain an infinity of experiences, one would never come to a *probability* which is confounded with moral certitude, that which would be concluding that experience is useless, & that the past proves nothing for the future.

Mr. Bernoulli, the geometer who understood the best these sorts of calculations, has offered the objection & has given the response to it. One will find it in his book *de arte conjectandi* p. 4, in all his extent; a problem, according to him, as difficult as the quadrature of the circle. There he shows that the *probability* which was born from repeated experience, went always by increasing, & increased so, that it approached indefinitely to certitude. His calculus teaches us to determine (question proposed in a fixed manner) how many times it would be necessary to reiterate the experience in order to arrive to an assigned degree of *probability*. Thus, in the case of an urn full of a great number of white & black balls, one wishes to be assured by experience of the ratio of the whites to the blacks; Mr. Bernoulli finds that in order that it is one thousand times more probable that there are two blacks out of three whites as not every other supposition, it is necessary to have drawn from the urn 25,550 balls, & that, in order that this be two thousand times more probable, it was necessary to have made 31258 tests, finally, in order that this became seven thousand times more probable, it was necessary 36960 drawings. The difficulty & the length of the calculation do not permit the reporting here in entirety, one can see it in the book cited.

Thence it is demonstrated that the experience of the past is a principle of *probability* for the future; that we have good reason to expect some events conformed to those which we have seen arrive frequently, & moreover we have good reason to expect them anew. This principle received, one senses of what utility they would be in the questions of physics, of politics, & that which regards the common life, some exact tables which would fix on a long sequence of events the proportion of those which arrive in a certain fashion to those who arrive otherwise. The usages which one has drawn of the baptismal and mortuary registers are so great, that these should engage not only to perfecting them by noting, for example, the age, the condition, the temperament, the kind of death, &c. but also to make there of many other events, which one says very improperly

to be the effect of chance; it is thus that one can form some tables which would note how many fires arrive in a certain time, how many epidemics of maladies are felt in certain periods of time, how many ships perish, &c. this which would be very convenient to resolve an infinity of useful questions, & would give to the attentive young people all the experience of the old men.

It is quite understood that one will not give into the abuse, which is not too ordinary, to the proof of experience, that one will not establish on a small number of facts a grand *probability*, that we will not go so far as to oppose or to prefer even a feeble *probability* to a contrary certitude, that we will not give into the foibles of those players who take only the cards which have won or those which have lost, although it is evident, by the nature of the games of chance, that the preceding trials influence not at all on the following. Superstition however rightly more pardonable than so many of the others, which, on the slightest experience or on the least consequent reasoning, is introduced only too much into the flow of life.

To these two general principles of *probability*, we can join more particulars, such as *the equal possibility of many events, the knowledge of causes, testimony, analogy & hypotheses.*

1. When we are assured that a certain thing can happen only in a certain determined number of ways, & that we know or suppose that all these ways have an equal possibility, we can say with assurance that the *probability*, that it will happen in such fashion, is worth so much, or is equal to so many parts of certitude. I know, for example, that by casting a die at random, I bring forth surely either 1 point, or 2, 3, or 4, or 5, or 6. We suppose besides the die perfectly true, the possibility is the same for all the points. There are therefore here six equal probabilities, which altogether make certitude; thus, each is a sixth part of this certitude. This principle, entirely simple as it appears, is infinitely fecund; it is on it that are formed all the calculations which one has made & which one can make on the games of chance, on lotteries, on assurances, & in general on all the *probabilities* susceptible of calculation. There is question only of a grand patience & of a detail of combinations, in order to untangle the number of favorable events & the number of contraries. It is on this principle, joined to experience, that one determines the *probabilities* of human life, or of the times that a person of a certain age can probably be deluded with expectations of life; that which makes the foundation of the calculus of values of old age pensions, of tontines. *See the essays on the probabilities*

of human life, & the works cited at the end of this article. It is extended to the calculation of the pensions put on two or three payables to the last living, on possessions, ration pensions, on assurance contracts, wagers, &c.

I have said that this principle was employed when we suppose the divers cases equally possible. And indeed, it is only by assumption relative to our knowledge, limited that we say, for example, that all the points of a die can come equally; this is not when they roll in the dice box, the one who must be presented, has not already the disposition which, combined with that of the dice box, of the mat, or of the force & of the manner with which one casts the die, must make it surely to arrive; but all this being entirely unknown to us, we have no reason to prefer one point to another; we suppose them therefore all equally facile to arrive. However there can often be error in this assumption. If we wished to seek the *probability* to bring forth 8 points with two dice, this would make a greater fallacy, but to reason thus: with two dice, I can bring forth either 2, or 3, or 4, or 5, or 6, or 7, or 8, or 9, or 10, or 11, or 12 points; therefore the *probability* to bring forth 8, will be $\frac{1}{11}$ of certitude; because it would be supposed that these 11 points are equally facile to bring forth, this which is not true. The most simple calculus of the game of tric-trac teaches us that out of 36 equally possible outcomes with two dice, 5 give us the point of 8; the *probability* will be therefore of 5 out of 36, or $\frac{5}{36}$ of certitude, & not $\frac{1}{11}$.

This fallacy is avoided easily in the calculus of games, where it is easy to determine the equal or unequal possibility of events; but it is more hidden, & it is only too common in the more composite cases. Thus, many people complain to be quite unhappy, because they have not been able to obtain certain happiness which is fallen in part to some others; they suppose that it was equally possible, equally proper, that this wealth arrived to them, without wishing to consider that they were not in a position so advantageous, which they had for them only a favorable manner, while the others had many of them, so that it would have been a great happiness that this sole way took place, without saying that the events which they attribute to chance, are directed by an infinitely wise providence, who has all calculations, & who, by some reasons unknown to us, disposes some things in a manner much more appropriate than is the arrangement only our feeble knowledge or our passions would wish to put them.

After simple *probability* comes a composite *probability* which depends yet on the same principle. This is the *probability* of an event which can happen only in the case that another event, itself simply probable, arrives. An example just to explain it. I suppose that in a game of quadrille of 40 cards, one demands of me to draw a heart, the *probability* of succeeding is $\frac{1}{4}$ of certainty, because, there are 4 colors & 10 cards of each color equally possible. But if one says to me next that I will win if I bring forth the king of hearts, then the *probability* becomes composite; because 1° it is necessary to draw a heart, & the *probability* is $\frac{1}{4}$; 2° supposing that I have drawn a heart, the *probability* will be $\frac{1}{10}$, because there are 9 other hearts that I can also rightly draw than the king. This *probability* enters on the first, only the tenth of a quarter, or the $\frac{1}{4}$ of $\frac{1}{10}$, that is to say $\frac{1}{40}$ of certitude. And it is clear, that since out of 40 cards I must draw precisely the king of hearts, I have of favorable only one case out of 40 equally possibles, or one against 39.

This composite *probability* is estimated therefore by taking from the first a part, such that one could take it away from entire certainty, if this *probability* was converted to certitude. A friend is departed for the Indies, in a fleet of twelve vessels, I understand that three of them have perished, & that the third of the crew of the rescued vessels is dead in the voyage; the *probability* that my friend is on one of the vessels arrived to a good port is $\frac{9}{12}$, & the one that he is not of the third dead en route, is $\frac{2}{3}$. The composite *probability* that he is yet alive, will be therefore $\frac{2}{3}$ of $\frac{9}{12}$ or $\frac{6}{12}$ or a half certitude. He is therefore for me between alive & dead.

We can apply this calculus to all sorts of proofs or reasonings, reduced for more clarity to the form prescribed by the art of reasoning: if one of the premises is certain, & the other probable, the conclusion will be the same degree of *probability* as this premise; but if the one & the other are simply probables, the conclusion will be only a *probability* of a *probability*, which is measured by taking from the *probability* of the major, a part such that expresses the fraction, which measures the *probability* of the minor. In these last examples the $\frac{9}{12}$ of $\frac{2}{3}$, which is the *probability* of the major, & the value of the conclusion will be $\frac{6}{12}$ or $\frac{1}{2}$.

Whence it would appear that the *probability* of the *probability* makes only

a very slight *probability*. What will be therefore a *probability* of the third or fourth degree? or what to think of those reasonings so frequent, of which the conclusion is based only on many probable propositions which must each be true in order that the conclusion be also? but if it sufficed that one alone of among them took place in order to verify the conclusion, it would be entirely the contrary; the more we stack the *probabilities*, the more the thing would become probable. If, for example, someone said to me, I give to you a louis if you bring forth with two dice 8 points, the *probability* of bringing forth 8 is $\frac{5}{36}$; if he added, I give to you yet if you bring forth 6: then as for winning, it suffices to bring forth one or the other, my *probability* will be $\frac{5}{36}$ & $\frac{5}{36}$, that is to say, $\frac{10}{36}$, that which increases my expectation to win.

Here are the elements on which we can determine all the questions, & the examples depending on this first principle of *probability*.

2. We pass to the second, which is the knowledge of the causes & of the signs, which we can regard as some causes or some occasional effects. We say only one particular word to the *probabilities*, referring for the rest to the *article* Cause. There are some causes of which the existence is certain, but of which the effect is only doubtful or probable; there are some others of which the effect is certain, but of which the existence is doubtful; there can be of them finally, of which the existence & the effect have only a simple *probability*. This distinction is necessary: an example will explain it. A friend has responded not at all to my letter; I seek the cause of it, there occur three of them: he is lazy, perhaps he is dead, or his affairs have prevented him from responding to me. He is lazy, first cause of which the existence is certain: I know that he writes with much difficulty; but the effect of this cause is uncertain, because a lazy man is determined sometimes to write. He is dead, second cause very uncertain, but of which the effect would be quite certain. He has some affairs, third cause uncertain in itself: I suspect only that he has many affairs, & of which the existence even supposed, the effect would be still uncertain, because one can have some affairs & find however the time to write.

The same thing must apply to the signs; their existence can be doubtful, their signification uncertain; & the existence & the signification can have only the credible. The barometer descends, this is a sign of rain of which

the existence is certain, but of which the signification is doubtful: the barometer descends often without rain.

From this distinction it follows that the conclusion drawn from a cause or from a sign of which the existence is certain, has the same degree of *probability* as is found in the effect of this cause, or in the signification of this sign. We have only to reduce the example of the barometer to this form. If the barometer descends, we will have rain: this is only probable; but the barometer descends, this is certain: therefore we will have rain; conclusion probable, of which experience gives the value. Likewise if the existence of the cause or of the sign is doubtful, but if its effect or signification is not, the conclusion will be the same degree of *probability* as the existence of the cause or of the sign. That my friend is dead, this is doubtful; the conclusion that I will draw from it, that I cannot write to me, will be equally doubtful.

But when the existence & the effect of the cause are probables, or if it is the question of signs, when the existence & the signification of the sign are only probables, then the conclusion has only a composite *probability*. We suppose that the *probability* that my friend has some affairs is $\frac{3}{4}$ of certitude, & this that his affairs, if he has them, prevent him writing to me is $\frac{2}{3}$ of this certitude, then the *probability* that he will not write to me, will be composed of two others, that which will be a half certitude.

3. We have indicated testimony as a third source of *probability*; & it keeps so closely to the subject of which we give the principles, that one can dispense with reporting here that which there is to say of it relative to the *probabilities* & to moral certainty. We can each see by ourselves: there are an infinity of things, often the most interesting, on which it is necessary to be related to the testimony of others. It is therefore important to determine, if it is not correct, at least in a manner which approaches it, the degree of agreement which we can give to this testimony, & which is for us the *probability* of it.

When one tells us a tale, or when one advances a proposition of a number of those which are substantiated by testimony, one must first examine the nature itself of the thing, & next weigh the authority of the testimony. If, on all sides, we find that it lacks no conditions requisite for the truth of the

proposition, we can not refuse him its consent; if it is evident that it lacks one or many of these conditions, one must not hold undecided to reject it; finally, if one sees clearly the existence of some of these conditions, & if we remain uncertain on the others, the proposition will be probable, & so much more probable, as a greater number of these conditions will hold.

1. As for the nature of the thing, the sole requisite condition, is that it be possible, that is to say, that there is nothing in its nature which forbids its existence, & nothing consequently which must prevent me from believing as soon as it will be sufficiently proved by an exterior proof, such as is that of testimony. To the contrary, if the thing is impossible, if it has in itself an invincible repugnance to exist, to some degree of credibility which can rise besides the proofs of the testimony, or other reasons extrinsic to its existence, I cannot believe it. Would someone claim to advance a contradiction, an absolute impossibility, he would join all sorts of proofs, he will never come to the end to persuade me that which is metaphysically impossible. A squared circle can be neither understood nor accommodated. Is the question a physical impossibility? we will be a little less difficult; we know that God has established himself the laws of nature, that he is constant in the observation of these laws; thus the mind repels to believe that they can be violated. However we know also that the one who has established them has the power to suspend them; that they are not an absolute necessity, but solely of convenience. Thus we ought not absolutely refuse our trust in testimonies or in the exterior proofs to the contrary; but it is necessary that these proofs be quite evident, in great number, & dressed in all the characters necessary in order to give our consent. Is it a question of moral impossibility or of an opposition to the moral qualities of intelligent beings? Although quite less delicate on the proofs or the testimonies which they wish to persuade us, however it is necessary that we see this credibility which is found in the same characters, & in the effects which result from them; it is necessary that the actions follow naturally from principles which ordinarily produce them: it is thus that it seems impossible that a wise man, of a grave & modest character, is carried without reason, without motive to commit an indecency in public. On the contrary, an ordinary morally possible fact, conformed to the regular course of nature, persuades easily; it carries already to itself many degrees of *probability*; if the testimony adds to it in the least, he would become very probable. This *probability* will increase yet by the accord of a

truth with others already known & established; if the story which one is making to us is so well linked with history, that one would not know how to deny it without reversing a sequence of well established historical facts, by that itself it is proven; if to the contrary it cannot find its place in history with deranging certain grand known events, by that even this story is rejected. Why is the history of the Greeks & Romans regarded among us as much more credible than that of the Chinese? it is that it leaves us an infinity of monuments of each kind which have a relation so necessary, or at least so natural with this history, & that links it such to general history, that they multiply the proofs of it to infinity; instead that that of the Chinese has only little of the liaisons with the sequence of this general history which is known to us.

2. When one has weighed the proofs which are drawn from the nature itself of the thing, that one has recognized the possibility, & in some manner the degree of intrinsic *probability*, it is necessary to come to the validity itself of the testimony. It depends on two things; on the number of testimonies, & on the confidence which one can have in each of them.

For this which is of the number of testimonies, there is no person who senses that their testimony is so much more probable, as they are in so much greater number: one would believe likewise that it increases in *probability* in the same proportion as the number grows; so that two testimonies of an equal confidence would give a *probability* double of that of one, but one would be deceived. *Probability* grows with the number of testimonies in a different proportion. If one supposes that the first testimony gives me a *probability* which is carried to the $\frac{9}{10}$ of certitude, the second, which I suppose equally believable, would it add to the *probability* of the first $\frac{9}{10}$? no, because then their two reunited testimonies would make $\frac{18}{10}$ of certitude, or a certitude & $\frac{8}{10}$ more, this which is impossible. I say therefore that this second testimony will increase the *probability* of the first by $\frac{9}{10}$ on that which remains to go to certitude, & will grow thus the reunited *probability* to $\frac{99}{100}$, that a third will carry it to $\frac{999}{1000}$, a fourth to $\frac{9999}{10000}$, thus in sequence, approaching always more to certitude, without ever arriving entirely there: that which must not surprise, since whatever number of testimonies as one supposes, there must always remain the possibility of the contrary, or some degrees of *probability* quite small to the truth, that they are mistaken; here is the proof of it. When two

testimonies say to me a thing, it is necessary, in order that I am mistaken by adding faith to their testimony, the one & the other induces me to error; if I am sure of one of the two, little import that the other be believable. Now, the *probability* that one & the other deceive me, is a *probability* composed of two *probabilities*, that the first deceives, & the second deceives. That of the first is $\frac{1}{10}$ (since the *probability* that the thing is conformed in its relation is $\frac{9}{10}$); the *probability* that the second deceives me also, is again $\frac{1}{100}$: therefore the composite *probability* is the tenth of the tenth or $\frac{1}{100}$; therefore the *probability* of the contrary, that is to say that which one or the other say truly, is $\frac{99}{100}$.

One sees that I represent to me here the moral certitude as the term of a race course that the diverse testimonies which come to support it the one & the other, make me traverse. The first approaches me from a space, which has with all the license the same proportion as the force of his testimony has with the entire certitude. If his relation produces to me $\frac{9}{10}$ of certitude, this first testimony will make me $\frac{9}{10}$ of the track. Comes the second testimony as believable as the first; it advances me on the remaining path, precisely as much as the first had advanced me on the total space: the one had brought me forth to $\frac{9}{10}$ of the course, the second approaches me again of $\frac{9}{10}$ of the remaining tenth; so that with these two testimonies I have made $\frac{99}{100}$ of all. A third of the same weight makes me travel through again $\frac{9}{10}$ of the remaining hundredth, between certitude & the point where I am; there will remain no more than the thousandth, & I will have made $\frac{999}{1000}$ of the course, & thus in sequence.

This method to calculate the *probability* of the testimony, is the same for a number of testimonies of which the credibility is different; that which, for the ordinary, is more conformed to the nature of things. Let a fact be rendered to me by three testimonies; the relation of the first is equivalent to $\frac{5}{6}$ of certitude; the second produces to me as $\frac{2}{3}$; & the third less believable than the two others, will give to me only a $\frac{1}{2}$ certitude if it were alone. Then, supposing always that I have no reason to suppose some concert among them, I say that their reunited testimony gives to me a *probability* that is $\frac{35}{36}$ of certitude, because the first advances me $\frac{5}{6}$, there remains $\frac{1}{6}$, of which the second will make me travel through $\frac{2}{3}$; thus there will be again $\frac{1}{3}$

of $\frac{1}{6}$, which is $\frac{1}{18}$; & the third advancing me $\frac{1}{2}$, I am no more extended from the end of the racecourse than $\frac{1}{36}$: I would have therefore traveled through $\frac{35}{36}$; besides it is indifferent in which order one takes them, the result is the same.

2. This principle can suffice for all the calculations on the value of testimony. As for the faith that each testimony merits, it is based on its *capacity* & on its *integrity*. By the first, it can not be deceived; by the second, it seeks not to deceive me; two equally necessary conditions; the one without the other does not suffice. Whence it follows that the *probability* that gives birth to the relation of a testimony in which we recognize this capacity & this integrity, must be regarded & calculated as a composite *probability*. A man comes to me to say that I have the big prize; I know him to be not too intelligent; he himself can be deceived: all counted, I evaluate the *probability* of his capacity at $\frac{8}{9}$; but perhaps he takes pleasure in deceiving me. We pose that there are odds of 15 against 1 that he is of good faith, the *probability* of his integrity will be therefore $\frac{15}{16}$. I say that the assurance of his testimony or the composite *probability* of his capacity, & of his integrity, will be $\frac{8}{9}$ of $\frac{15}{16}$, that is to say $\frac{5}{6}$ of certitude.

The surest manner to judge the capacity & integrity of a testimony, would be *experience*. It would be necessary to know exactly how often this same man has deceived or has said the truth; but this experience is limited, & wanted for the ordinary. Instead one has recourse to the public & particular rumors, to the exterior circumstances where the testimony is found. Has he received a good education? is he of a rank which is supposed to engage him to respect further the truth? is he of an age which gives more weight to his testimony? is he disinterested in this? or what can be his end? extract some advantage from it, or shun thence some pain? his taste, his passion are they flattered to deceive us? is this a sequence of prevention, of hatred? Quite as much of circumstances which it is necessary to examine if we have not experience, & of which it is quite difficult to determine the correct value.

Moreover, the capacity of a testimony supposes, outside the well-conditioned sense, a certain firmness of the mind which allows oneself neither frightened by danger, nor surprised by novelty, nor seduced by a judgment too rash. It is more believable in proportion as the thing of which

we speak is more familiar & more known; his story itself makes often a proof of his capacity, & announces to me that he has taken or ignored all the necessary precautions in order to not deceive himself: the more he has reiterated them, the more he has the right to my confidence. This capacity to understand well depends yet on the attention to observe, on the memory, on the time: other conditions which, in the manner to relate clearly & in detail, influences on the degree of *probability* which a testimony merits.

One must not ignore the silence of those who would have interest to contradict a testimony, if at least he is extorted neither by fear, nor by authority. It is difficult indeed to estimate the weight of a similar negative testimony; one can assure in general that the one who makes simply only to be quiet, merits less attention than the one who assures a fact. If nonetheless the fact is such that he has been able to ignore it, if he would have served to evaluate the rest of his story, if he would have been interested in the relation, or if his duty would call it; in a similar case, it is certain that his silence is worth a testimony, or at least it reduces & diminishes the *probability* of the opposing testimony.

We must still say a word on the testimonies by hearsay, or on the diminishing of testimony which, passing from mouth to mouth, they arrive to us only by way of a chain of testimony. It is clear that hearsay testimony, all things equal besides, is less believable than an eyewitness testimony; because if the one is deceived, or has wish to deceive, the hearsay testimony which follows it, however faithful, will report to us only an error; & even then the first would have debited the truth, if the hearsay testimony is not faithful, if he has wrong intended, if he has forgotten or confounded some essential part of the story, if he mixes with his own, he no longer reports the pure truth; thus, the confidence that we owe to this second testimony, is reduced already, will be reduced in measure as it will pass by more mouths, in measure as the chain of testimony will be extended. It is easy to calculate on the established principles, the value of this *probability*.

We follow the example of which we have made use. Pierre announces to me that I have had a lot of one thousand livres: I estimate his testimony to $\frac{9}{10}$ of certitude, that is to say that I will give my expectation only for 900

francs. But Pierre says to me that he knows it from Jacques; now, if Jacques would have said to me, I would have estimated his report to $\frac{9}{10}$ by supposing it as believable as Pierre; thus me who am not entirely sure that Pierre is not deceived in receiving this testimony of Jacques, or that he has not some design to deceive me, I must count only $\frac{9}{10}$ of 900 livres, or on $\frac{9}{10}$ of $\frac{9}{10}$ of 1000 livres, that which makes 810 livres. If Jacques would hold the fact of another, I would take again on this last assurance $\frac{9}{10}$, supposing this third equally believable, & my expectation would be reduced to $\frac{9}{10}$ of $\frac{9}{10}$ of 1000 livres, or 729 livres, & thus in sequence.

Who will wish to take the pain to calculate on this method, will find that, if the confidence that one must have in each testimony is of $\frac{95}{100}$, the thirteenth testimony will transmit no more than $\frac{1}{2}$ certitude, & then the thing will cease to be probable, or there will not be more extrinsic reason to believe it, than to not believe it. If the *probability* due to each testimony is $\frac{99}{100}$, it will be reduced to $\frac{1}{2}$ certitude only when the testimony will have passed by seventy mouths; & if this confidence were suppose of $\frac{999}{1000}$, it would be necessary a chain of 700 testimonies in order to render the fact uncertain.

These long enough calculations can be shortened by this general rule, of which the simple algebra furnishes us the result & the demonstration. Take $\frac{7}{10}$ of the quotient of the division of the *probability* of a simple testimony by the contrary *probability*, as here of $\frac{95}{100}$ by $\frac{5}{100}$, or of 95 by 5, which is 19, of which I take $\frac{9}{10}$, & you will have the testimony which leaves you in a half-certitude; in this example this is $13\frac{3}{10}$, that which gives the thirteenth testimony.

It will be likewise if the successive testimonies are supposed of unequal force; whence there is place to conclude in general, that it is necessary to make little basis on hearsay, without leaving to go however to historic Pyrrhonism, since here one can reunite the *probabilities* that give many collateral chains of successive testimonies. We suppose that a fact arrives to us by a simple succession of testimonies by word of mouth, in a manner that each testimony succeeds to the other at the end of twenty years, & that the confidence to each testimony diminishes by $\frac{9}{10}$; by the preceding rule, at the end of twelve successions, or of 240 years, the fact would

become uncertain, being proved only by these 12 testimonies; but, if this chain of testimonies is fortified by nine other similar chains which combine to attest to the same truth, then there will be more than one thousand against one in favor of the fact; if one supposes one hundred chains of testimony, there will be more than two million against one in favor of the fact.

If the testimony is transmitted by writing, the *probability* increases infinitely, as long as it subsists & is conserved a quite long time; the concurrent testimony of many copies or printed books which form as many different chains, give a *probability* so great as it approaches indefinitely certitude; because to suppose that each copy can endure 100 years, that which is the less, & if at the end of this time the authority, not of a single copy, but of all those which have been made on the same original, is alone $\frac{99}{100}$, then it will be necessary more than seventy successions of 100 years, or 7000 years in order that the fact become uncertain; & if one supposes many chains of testimonies, which combine all to attest the same fact, the *probability* increases so strong, that it becomes infinitely little different from entire certitude, & will surpass by much the assurance which one could have of the mouth of one or even of many eyewitness testimonies. There are other circumstances which it is easy to suppose & which demonstrate the great superiority of the written tradition over the oral tradition.

We have indicated two other sources of *probability*, analogy & hypotheses on which we refer to the *articles* INDUCTION, ANALOGY, HYPOTHESIS, SUPPOSITION. These principles can suffice to explicate all the theory of *probability*. We have given only the elements; one will find the application of it in all the good works, which are in great number on this subject. Such are the *Essais sur les probabilités de la vie humaine*, by Mr. Deparcieu; *l'Analyse des jeux de hasard*, by Mr. de Montmort, who gives the theory of combinations, for example the *article* of this Dictionary under this word, & many others which have relation, especially the *Ars Conjectandi*, of Mr. Jacques Bernoulli, & some *Memoirs* of Mr. Halley, which are found in the transactions of England⁴, n. 196 & the following, which all serve to determine the truth of the events, & the degrees by which we arrive to

⁴ Philosophical Transactions of the Royal Society.

moral certitude.

We conclude that it would not be entirely impossible to reduce all this theory of *probabilities* to a regular enough calculus, if good geniuses would wish to combine by the researches, some observations, a follow-up study, & an analysis of the heart & of the spirit, based on experience, to cultivate this branch so important of our knowledge, & so useful in the continual practice of life. We agree that there is yet more to make, but the consideration of this which misses must excite to fill these guides, & the importance of the object offers of what to compensate amply some difficulties.

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