

**EXTRACTS FROM THREE EULOGIES OF JAKOB BERNOULLI  
REGARDING THE CONTENTS OF THE *ARS CONJECTANDI***

FONTENELLE, JACOB HERMANN & JOSEPH SAURIN

Extract from Letter 86 of Pierre Varignon to Johann Bernoulli  
6 September 1705

*Die Werke von Jacob Bernoulli*, Vol. 3, p. 392.

“I await always the Memoirs that I have demanded to Mr. Hermann on the part of Mr. de Fontenelle in order to make the funeral Eulogy of Mr. your brother: It is necessary they be the most ample and the most detailed that he could make them, from his youth to his death. This Eulogy will be read publicly by Mr. de Fontenelle in the assembly on next Saturday 14 November, on return from our vacations which began yesterday, which assembly is always very numerous, the door being open to everyone there that day.”

Extract from the Eloge (by Fontenelle) of Jakob Bernoulli  
*Histoire de l'Académie Royale des Sciences*, 1705 pp. 139-150.

[148] “He achieved a great work *De Arte Conjectandi*, & although nothing of it has appeared, we are able to give an idea of it on the testimony of M. Herman. The Rules of a game being supposed, & two Players of the same strength, one is able, in whatever state that a game may be, to determine for the advantage that one of the Players has on the other, how much greater are the odds that he will win. The wager changes according to all the different states where the game will be, & when one wishes to consider all these changes, one finds sometimes some Series or sequences of regulated Numbers, & even new and singular. If one supposes the Players unequal, one demands what advantage the strongest must accord to the other, or reciprocally the one having agreed with the other a certain advantage, one demands by how much he is stronger, & it is notable how often the advantages or the forces are incommensurables, so that the two Players are never able to be perfectly equal. The reasonings that these kinds of matters demand are ordinarily more delicate, more refined, composed of a greater number of views which are able to be overlooked, & consequently more subject to error than other mathematical reasonings. For example, two equal Players playing to 4 linked games, if one has won 3 of them & the other 2, it is necessary to reason correctly enough in order to determine precisely that one can wager 3 for the one who has the 3 games, & 1 only for the one who has 2 of them. This case is the simplest, & one is able to judge thence of those which are infinitely more complicated. Some great Mathematicians, & principally Messers Pascal & Huygens, have already proposed or resolved some Problems on this matter, but they have done only to touch it lightly, & Mr. Bernoulli embraced it in a greater extent, & studied it thoroughly much further. He carried it even to Moral & Political things, & it is there that the Work must have more new, & more surprise. However if one considers closely the things of life on which one has daily to deliberate, one will see that the deliberation must be reduced, as the

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Translated by Richard J. Pulskamp, Department of Mathematics and Computer Science, Xavier University, Cincinnati OH. August 16, 2009.

Wagers that one would make on a game, by comparing the number of cases where a certain event will arrive to the number of cases where it will not arrive. This done, one could know correctly, & one could express by some numbers by how much the part that one could take will be the better. All the difficulty is that many cases escape us where the event can arrive, or not arrive, & the more there are of these unknown cases, the more the knowledge of the part that one must take would appear uncertain. The sequence of these ideas has led Mr. Bernoulli to this question, If the number of unknown cases diminishing always the probability of the part that one must take of it increases necessarily, so that it comes at the end to such degree of certitude as one will wish. It seems that there is no difficulty for the affirmation of this Proposition, however Mr. Bernoulli who possessed well this matter ascertained that this Problem was much more difficult than the one of the Quadrature of the circle, & certainly it would be without comparison more useful. It is not as glorious to the Spirit of Geometry to regulate in Physics, as in the Moral things, so complicated, so contingent, so changeable; the more a subject is opposite to it, & unyielding, the more there is honor to master it.”

Extract from the Eloge (by Joseph Saurin) of M. Bernoulli,  
previous Professor of Mathematics at Basel.  
*Journal des Sçavans* 1706, pp. 81-89.

“M. Bernoulli had worked much on numbers, & especially he had much studied the matter of *permutations* & of *combinations*. He had need of this knowledge in a Work which he planned, & which he had nearly achieved when he died. The title of the Work must be, *De Arte conjectandi*, On the Art of conjecturing. The Author determines in effect, & reduces to the calculus, the different degrees of certitude or of probability of the conjectures that one can form on the things which depend on chance; this which he extends even to civil life & to particular affairs. Among the *Miscellanea* which are at the end of the *Exercitationes* of Mathematics of Franz Schooten, one finds a little treatise of Mr. Huygens, *De Ratiociniis in ludo aleae*; that is to say, on the Reckonings which take place in the games of chance. Mr. Huygens gives a method in order to determine accurately by the calculus, the different lot of the players in diverse cases which present themselves; he resolves actually many of them, & he introduces the operation & the demonstration of it; but at the end of the Treatise, he leaves some of them of which he gives simply the determination, without indicating the way that he has followed, & without any demonstration. The Work of Mr. Bernoulli is divided into four parts, of which the first contains this little Treatise of Mr. Huygens, with some Remarks of Mr. Bernoulli, who among the cases you leave without analysis & without demonstration, he resolves those which can be resolved without the help of *permutations* & of *combinations*. This matter of permutations & of combinations is treated at foundation in the second part, where among a great number of considerable Problems, one finds the one resolved in a general manner: *On the numbers being in Arithmetic progression, to find the sum of their powers*. The third part shows the usage of the doctrine of *permutations* & of *combinations* for the resolution of the questions which one can propose in the games of chance, & in some other subjects which depend likewise on fate. The cases of Mr. Huygens which remained to be solved, are resolved. It is in the fourth part that the Author extends his Method, & his reasonings, thus as one has said, to the things which regard civil life, & domestic affairs. The foundation of this last part, is an important Problem, that he resolves first, a solution of which he made a greater event, than the Quadrature of the Circle. It concerns determining if by increasing the number of observations, with respect to an event, one increases also at the same time in proportion the

degree of probability or of appearance that there is found the true ratio between the number of cases where the event can arrive, & the number of cases where it can not arrive; so that one can finally attain to a degree of probability or of appearance which is above any given degree; that is to say which is a veritable certitude.

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“There is place to hope that some friendly & able hand will add to a Treatise so curious that which can be lacking: but while one could give it such as it is, it will do always much to please to the Public, especially if, with the universal Gnomonic Tables, which one says to be in state to see the day, one joins there that which one will find among the papers of this illustrious Geometer, the more dignity of his reputation & of the curiosity of the Scholars. This is that which they await in particular from the care of the young Mr. Bernoulli, who has always made appear a great zeal for public utility, & for the advancement of the Sciences, & to whom the memoir of one such brother must be dear.”

Extract from the Eulogy of Jakob Bernoulli by Hermann<sup>1</sup>

Attachment to Letter XIII of 28 October 1705 from Hermann to Leibniz.

*Leibnizens Mathematische Schriften*, Bd. IV, pp. 285-292

“He always intended the *Ars Conjectandi* should see the light of day, he barely had written it out to the center when premature death seized him; in it he teaches to apply the art of reckoning in games of chance to morals, civics and economics, at the end he solved a certain singular Problem, with respect to which first in extent of utility and then in difficulty of invention he placed by far before the Quadrature of the circle, which if it were the greatest thing he discovered, is but of little benefit.”

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<sup>1</sup>This Eulogy was to have been printed in the *Acta Eruditorum* 1706. The Eulogy may be found on pp. 41-45.