Mémoire sur les suites récurro-récurrentes et sur leurs usages dans la théorie des hasards.

P.S. de la Place

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PROBLEM IV. — A lottery being composed of a number n of numerals 1, 2, $3, \ldots, n$, of which there is extracted a number p at each drawing, one demands the probability that after x drawings all the numbers will be extracted.

We suppose that S wagers that all the numbers will not be extracted after this number of drawings, and we seek all the cases favorable to S; it is clear that their number is equal:

1. To the number of cases following which the number 1 is not able to be extracted after the drawing x;

2. To the number of cases following which the number 2 is not able to be extracted, the number 1 being extracted;

3. To the number of cases following which the number 3 is not able to be extracted, the numbers 1 and 2 being extracted, and thus in sequence; if therefore one names ${}_qy_x$ the sum of all these cases to the numeral q, one will have

$$ {}_qy_n = {}_{q-1}y_n - {}_{q-1}y_{n-1} + \left[\frac{(n-1)\cdots(n-p)}{1.2\ldots p}\right]^x, $$

equation which refers back to Problem I, q and n being supposed variables and x constant; here is how one can integrate in this particular case; putting q successively equal to 1, 2, 3, $\ldots,$ one will have

$$ {}_1y_n = \left[\frac{(n-1)\cdots(n-p)}{1.2\ldots p}\right]^x, $$

$$ {}_2y_n = 2\left[\frac{(n-1)\cdots(n-p)}{1.2\ldots p}\right]^x - \left[\frac{(n-2)\cdots(n-p-1)}{1.2\ldots p}\right]^x, $$

$$ {}_3y_n = 3\left[\frac{(n-1)\cdots(n-p)}{1.2\ldots p}\right]^x - 3\left[\frac{(n-2)\cdots(n-p-1)}{1.2\ldots p}\right]^x + \left[\frac{(n-3)\cdots(n-p-2)}{1.2\ldots p}\right]^x, $$

whence one will conclude easily

$$ {}_n y_n = n\left[\frac{(n-1)\cdots(n-p)}{1.2\ldots p}\right]^x - \frac{n(n-1)}{1.2}\left[\frac{(n-2)\cdots(n-p-1)}{1.2\ldots p}\right]^x + \frac{n(n-1)(n-2)}{1.2.3}\left[\frac{(n-3)\cdots(n-p-2)}{1.2\ldots p}\right]^x + \cdots $$

Now here the sum of all the possible cases is $\left[\frac{n(n-1)\cdots(n-p+1)}{1.2\ldots p}\right]^x$; naming therefore z_x the probability of S, one will have

$$ z_x = n\left[\frac{(n-1)(n-2)\cdots(n-p)}{n(n-1)\ldots(n-p+1)}\right]^x - \frac{n(n-1)}{1.2}\left[\frac{(n-2)\cdots(n-p-1)}{n\ldots(n-p+1)}\right]^x + \cdots $$

If one wishes to apply this formula to the lottery of the École militaire, it is necessary, following the nature of this lottery, to suppose $n = 90$ and $p = 5$.