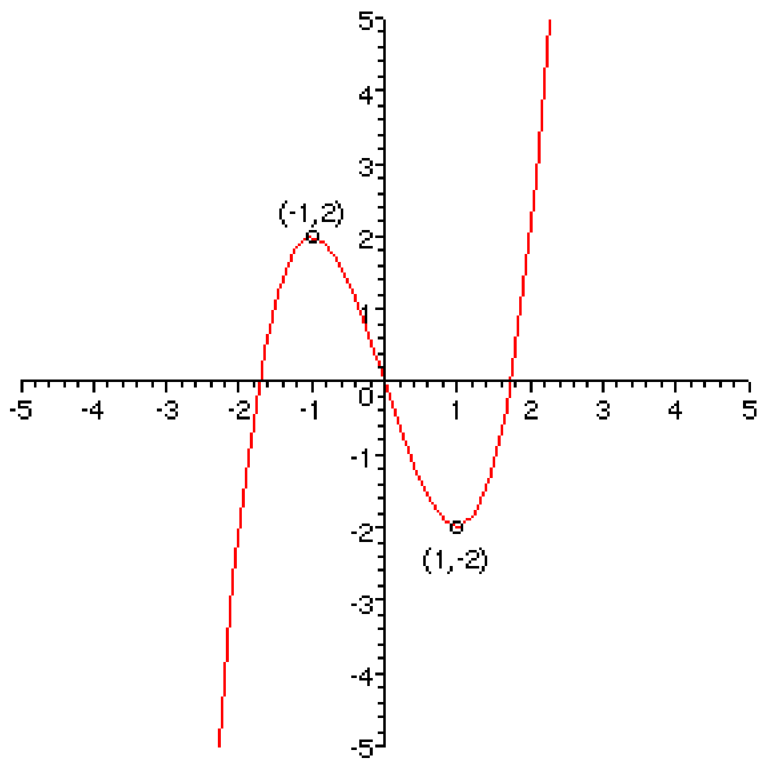
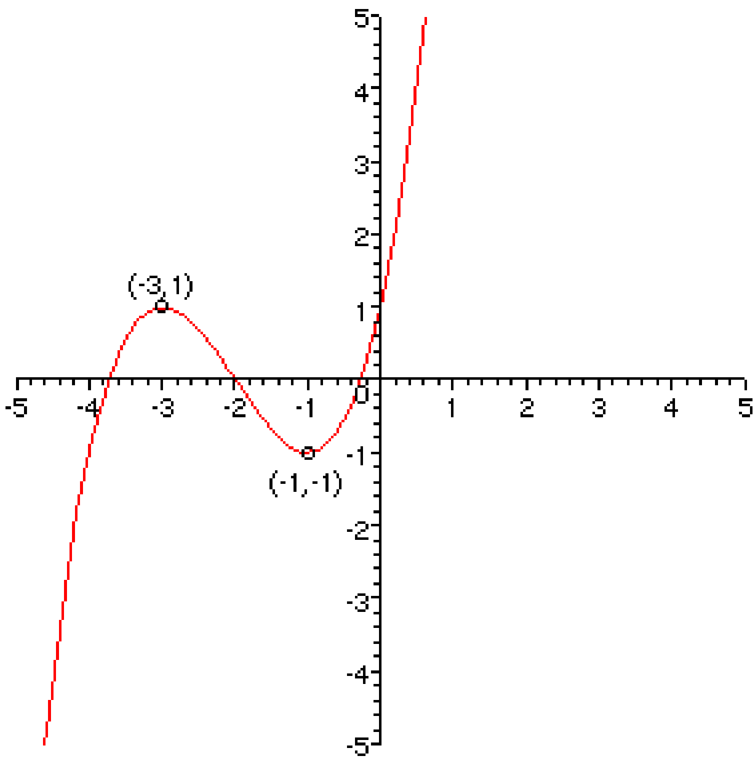
There are two forms of the test; both are included below.

1. [20 points] On the axes on the left, graph the function $f(x) = x^3 - 3x$. Label the coordinates of the points on the graph at $x = \pm 1$. Then on the axes on the right, graph the transformed function $g(x) = (1/2)f(x+2)$ and label the corresponding points on this curve.





2. [15 points] The *Cincinnati Enquirer* (October 10, 1998) reported that the number of personal bankruptcies was on the rise in the late 1990's. A good model that fits their data is given by the quadratic function $B(t) = 321t^2 - 2514t + 10442$. Here, t measures years since 1990 and $B(t)$ counts the number of personal bankruptcies in the Tristate area in year t. Find the vertex of the parabola which is the graph of the function $B(t)$. What does it tell about the number of bankruptcies?

The vertex has x-coordinate $h = -b/2a = -(-2514)/(2*321) = 3.9$ and y-coordinate $k = B(h) = 5519.7$. So the vertex is the point (3.9, 5519.7). This means that 3.9 years after 1990 (in 1994) bankruptcies in the Tristate hit an annual low of roughly 5520.

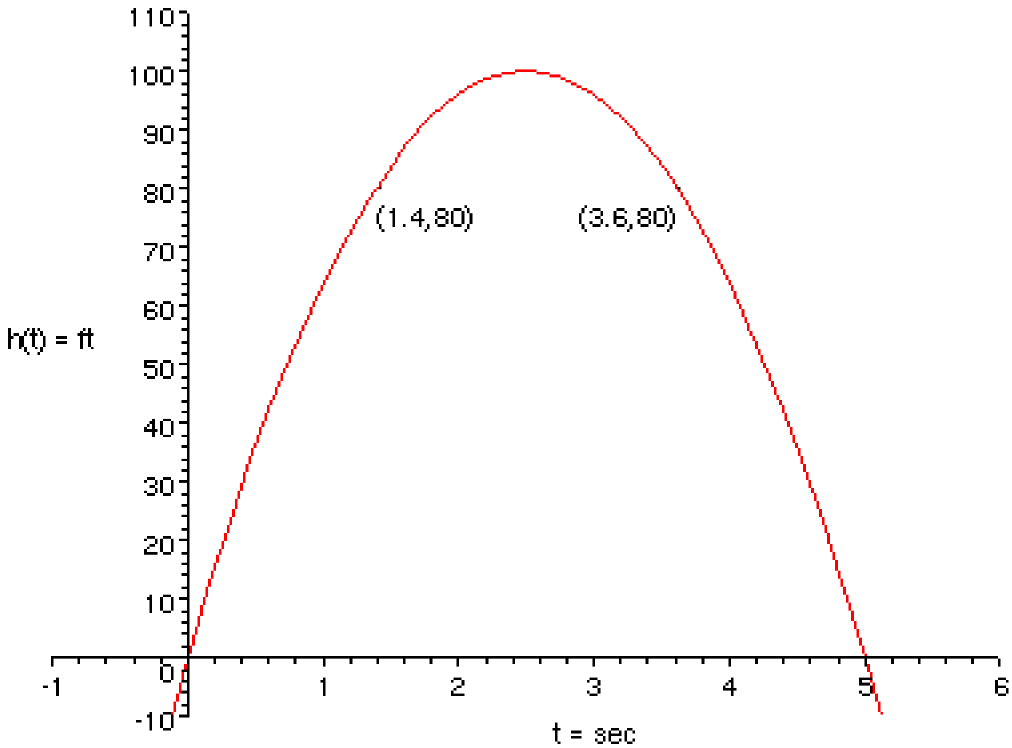
3. [15 points] A ball is thrown into the air. Its height in feet t seconds later is given by the equation $h(t) = 80t - 16t^2$. Solve the equation $h(t) = 80$. Interpret your solutions and illustrate them on a graph of $h(t)$.

We solve:

$$80 = 80t - 16t^2$$
$$16t^2 - 80t + 80 = 0$$
$$t^2 - 5t + 5 = 0$$

(quadratic formula:) $t = \dfrac{5 \pm \sqrt{5^2 - 4 \cdot 5}}{2} = \dfrac{5 \pm \sqrt{5}}{2} = 1.4, 3.6$

So the ball reaches a height of 80 ft twice: on the way up at $t = 1.4$ sec, and on the way down at $t = 3.6$ sec.



4. (a) [10 points] If $u(v(x)) = w(x) = \dfrac{3}{1+\sqrt{x}}$, find possible formulas for $u(x)$ and $v(x)$.

There are many ways to answer this problem. Here are two different correct answers: $u(x) = \dfrac{3}{x}$ and $v(x) = 1 + \sqrt{x}$; or $u(x) = \dfrac{3}{1+x}$ and $v(x) = \sqrt{x}$.

(b) [10 points] Give a formula for $w^{-1}(x)$.

An equation for the inverse will reverse the roles of input and output, and solve for y:

$$x = \frac{3}{1+\sqrt{y}}$$

$$x(1+\sqrt{y}) = 3$$

$$1 + \sqrt{y} = \frac{3}{x}$$

$$\sqrt{y} = \frac{3}{x} - 1$$

$$y = \left(\frac{3}{x} - 1\right)^2$$

5. [15 points] Complete the table.

x	$f(x)$	$g(x)$	$f(g(x))$
1	–1	3	5
2	4	1	**–1**
3	**5**	2	**4**

Here's why: $f(g(2)) = f(1) = -1$; $f(3) = f(g(1)) = 5$; $f(g(3)) = f(2) = 4$.

6. The use of automatic teller machines is continuing a steady growth in the U.S. Using data from the mid-1990's (from the August 31, 1997, *Cincinnati Enquirer*), a model tracking the annual number of transactions at ATMs nationwide (in millions) is given by $A(t) = 61.4t + 521.9$ where t measures the number of years since 1990. The same report yields the model formula $p(t) = 0.0065t^2 - 0.0553t + 0.682$ for the percentage of ATM transactions in which customers withdrew cash from their accounts.

   (a) [5 points] Calculate $A(5)$ and $p(5)$ and interpret these numbers.

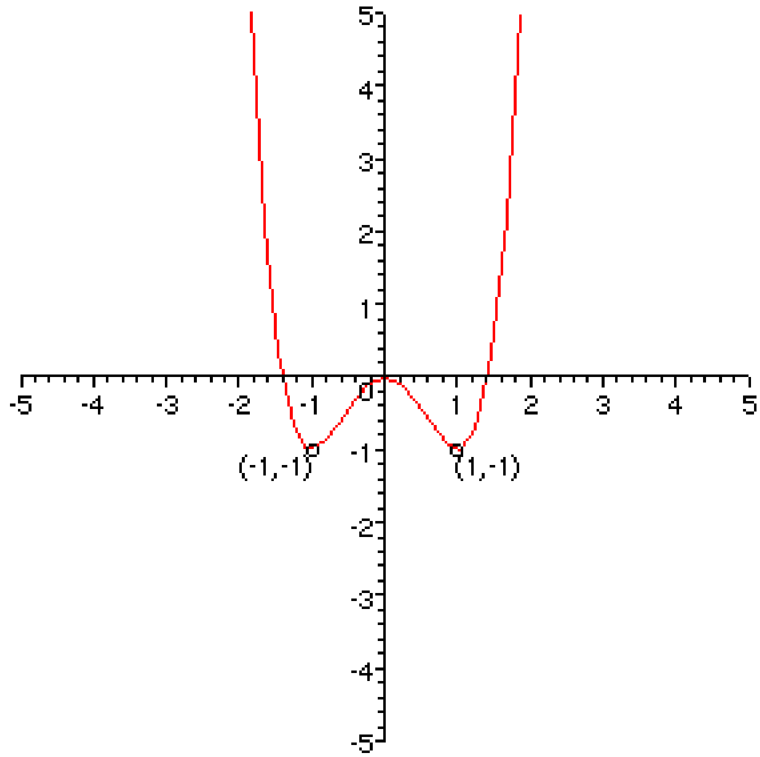
$A(5) = 61.4 \cdot 5 + 521.9 = 828.9$ million transactions at ATMs in 1995
$p(5) = 0.0065 \cdot 5^2 - 0.0553 \cdot 5 + 0.682 = .568 = 56.8\%$ of ATM transactions were cash withdrawals in 1995

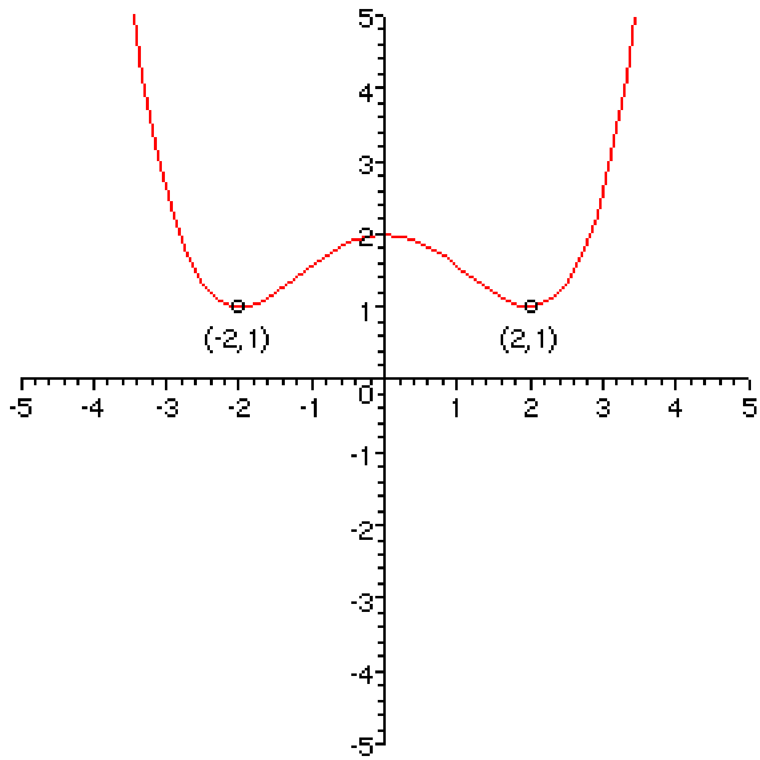
   (b) [10 points] Let $f(t) = A(t) \cdot p(t)$ be the function obtained by multiplying the functions A and p. What does the function $f(t)$ measure? Calculate and interpret the value of $f(5)$.

$f(t)$ = # of transactions at ATMs nationwide (in millions) which were cash withdrawals.
Therefore, $f(5) = 470.8$ million ATM transactions in 1995 were cash withdrawals.

This is the second form of the test.

1. [20 points] On the axes on the left, graph the function $f(x) = x^4 - 2x^2$. Label the coordinates of the points on the graph at $x = \pm 1$. Then on the axes on the right, graph the transformed function $g(x) = f(0.5x) + 2$ and label the corresponding points on this curve.





2. [15 points] The practices of fishermen worldwide has been a concern of environmentalists for years. Data from the early 1990's yields the following quadratic function as a model for the size of catches of fish (in millions of metric tons) measured in year t after 1990: $F(t) = 1.155t^2 - 1.743t + 97.462$. For instance, in 1993, total world catches of fish amounted to $F(3) = 102.6$ million metric tons. Find the vertex of the parabola which is the graph of the function $F(t)$ and explain what it tells us about world fishing.

The vertex has x-coordinate $h = -b/2a = -(-1.743)/(2*1.155) = .75$ and y-coordinate $k = B(h) = 96.804$. So the vertex is the point (.75, 96.8). This means that .75 years after 1990 (by 1991) 96.8 million metric tons were fished worldwide, representing a minimum value, and (since $a = 1.155$ is positive) this total has been growing ever since.

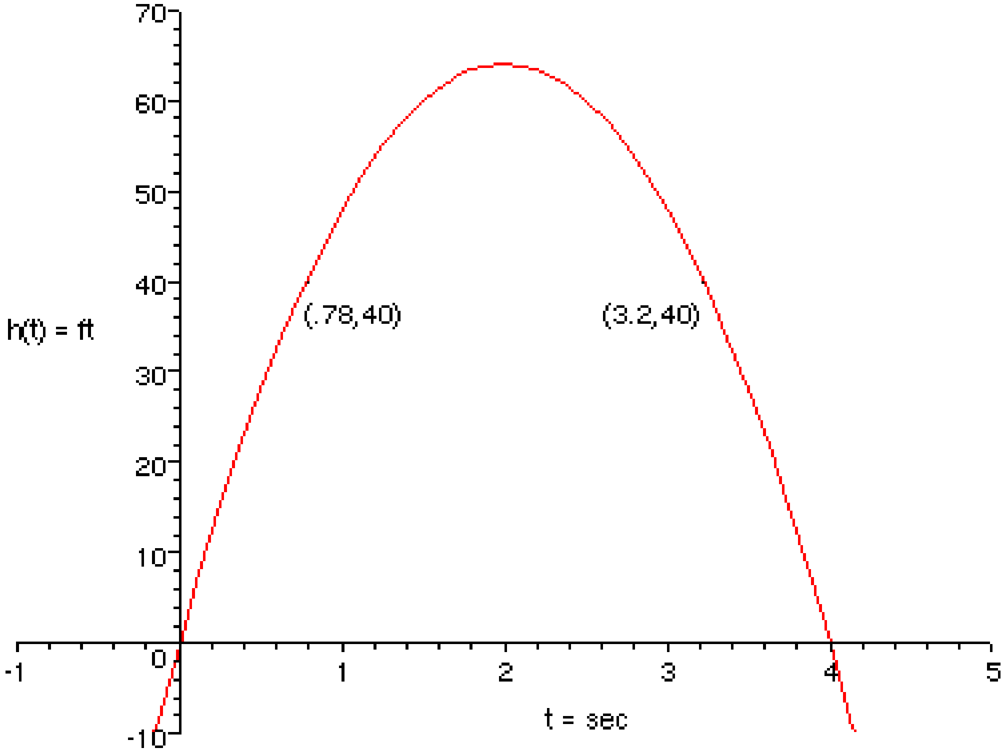
3. [15 points] A ball is thrown into the air. Its height in feet t seconds later is given by the equation $h(t) = 64t - 16t^2$. Solve the equation $h(t) = 40$. Interpret your solutions and illustrate them on a graph of $h(t)$.

We solve:

$$40 = 64t - 16t^2$$
$$16t^2 - 64t + 40 = 0$$
$$2t^2 - 8t + 5 = 0$$

(quadratic formula:) $t = \dfrac{8 \pm \sqrt{8^2 - 4 \cdot 2 \cdot 5}}{2 \cdot 2} = \dfrac{8 \pm \sqrt{24}}{4} = .78,\ 3.2$

So the ball reaches a height of 40 ft twice: on the way up at $t = .78$ sec, and on the way down at $t = 3.2$ sec.



4. a) [10 points] If $u(v(x)) = w(x) = \dfrac{1}{\sqrt{x^2 + 1}}$, find possible formulas for $u(x)$ and $v(x)$.

There are many ways to answer this problem. Here are two different correct answers: $u(x) = \dfrac{1}{\sqrt{x}}$ and $v(x) = x^2 + 1$; or $u(x) = \dfrac{1}{x}$ and $v(x) = \sqrt{x^2 + 1}$.

(b) [10 points] Give a formula for $w^{-1}(x)$.

An equation for the inverse will reverse the roles of input and output, and solve for y:

$$x = \frac{1}{\sqrt{y^2 + 1}}$$

$$x\sqrt{y^2 + 1} = 1$$

$$\sqrt{y^2 + 1} = \frac{1}{x}$$

$$y^2 + 1 = \frac{1}{x^2}$$

$$y^2 = \frac{1}{x^2} - 1$$

$$y = \sqrt{\frac{1}{x^2} - 1}$$

5. [15 points] Complete the table.

x	$f(x)$	$g(x)$	$f(g(x))$
1	0	3	–4
2	5	1	**0**
3	**–4**	2	**5**

Here's why: $f(g(2)) = f(1) = 0$; $f(3) = f(g(1)) = -4$; $f(g(3)) = f(2) = 5$.

6. [10 points] Census figures for the early 1990's give data on the populations of each of the 50 states. The following exponential models give population functions for the states of Ohio, Kentucky and Indiana (where t measures years since 1990, and populations are measured in thousands):

$P_{OH}(t) = 10875(1.004825)^t$; $P_{KY}(t) = 3687(1.008985)^t$; $P_{IN}(t) = 5553(1.008674)^t$

(a) Determine the populations of each of the three states in 2001, based on these models. Also give the total population of the tristate area in 2001.

The population of Ohio in 2001 is $P_{OH}(11) = 11466.319$ thousand = 11,466,319
The population of Kentucky in 2001 is $P_{KY}(11) = 4068.225$ thousand = 4,068,225
The population of Indiana in 2001 is $P_{IN}(11) = 6106.421$ thousand = 6,106,421

So the total population of the tristate is 21,640,965.

(b) Write a formula for the function $P(t)$ which gives the total tristate population in year t. Calculate and interpret the quantity $P(11)$.

$P(t) = P_{OH}(t) + P_{KY}(t) + P_{IN}(t)$
$= 10875(1.004825)^t + 3687(1.008985)^t + 5553(1.008674)^t$

From this we find that the total population of the Tristate is
$P(11) = 21640.965$ thousand = 21,640,965.