

Form A

1. (a) [10 points] Use the data in the table below to create a cubic model for $D(t)$ = number of public high school graduates (in thousands) in the U.S. in year t after 1980. Give the formula with coefficients rounded to two decimal places.

Year	1980	1990	1995	1999
Thousands of graduates	2747.7	2320.3	2273.5	2502.8

The calculator gives $D(t) = 0.27t^3 - 4.60t^2 - 24.07t + 2747.70$ for the model.

- (b) [10 points] It is clear from the table that the number of graduates has been in decline until recently, when the numbers indicate an increase. Compute the formula for the first derivative of $D(t)$ and use it to determine the year in which the number of public high school graduates in the U.S. reached a minimum and what that minimum number was.

We find $D'(t) = 0.81t^2 - 9.20t - 24.07$; setting this equal to zero gives a quadratic equation whose solutions are

$$t = \frac{9.2 \pm \sqrt{9.2^2 - 4(0.81)(-24.07)}}{2(0.81)} = 13.55, -2.19$$

The first solution is the only one in the domain of interest, and gives the value of the maximum. In 1993, the minimum number of graduates was $D(13.55) = 2248.7$ thousand.

- (c) [10 points] Compute the formula for the second derivative of $D(t)$ and use it to determine the year in which the rate of decline in the number of high school graduates was greatest and what that greatest rate of decline was.

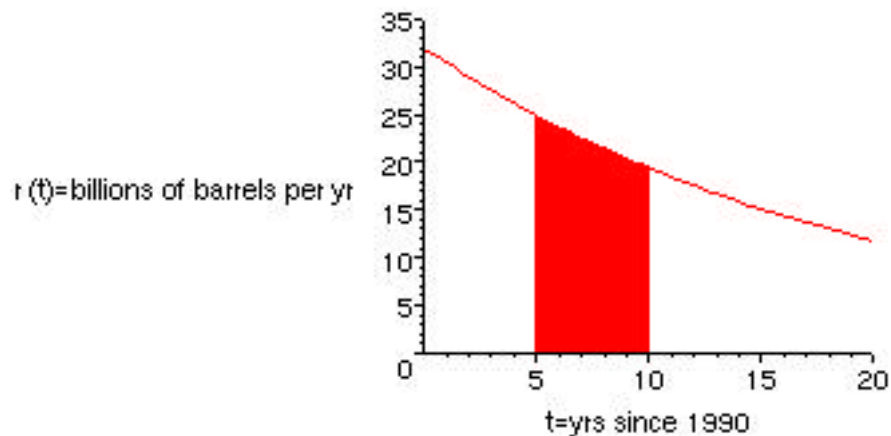
We find $D''(t) = 1.62t - 9.20$; setting this equal to zero leads to $t = 5.68$. That is, the rate of decline in the number of high school graduates was greatest in 1985, when they were declining at the rate of $D'(5.68) = -50.19$ thousand graduates per year.

Suppose that $r(t) = 32e^{-.05t}$ measures the rate (in billions of barrels per year) of consumption of world oil reserves, with t measuring time in years since 1990.

2. [10 points] Use the formula to complete the table (record values to one decimal place accuracy):

Year	1995	1996	1997	1998	1999	2000
Consumption of reserves	24.9	23.7	22.6	21.5	20.4	19.4

Then sketch a graph of this function, properly labeled, for the domain $0 \leq t \leq 20$ and shade the area between the t -axis and the graph between $t = 5$ and $t = 10$.



3. [10 points] Write the correct integral notation to represent the shaded area in your graph in #2. Then write a sentence describing what this quantity measures.

$\int_5^{10} 32e^{-.05t} dt$ = total consumption of oil reserves (in billions of barrels) between 1995 and 2000.

4. [10 points] Use your table values from #2 to give an estimate for the value of this integral by means of 5 trapezoids. Show your arithmetic.

$$\begin{aligned} & \int_5^{10} 32e^{-.05t} dt \\ & \approx \frac{24.9 + 23.7}{2} \Delta t + \frac{23.7 + 22.6}{2} \Delta t + \frac{22.6 + 21.5}{2} \Delta t + \frac{21.5 + 20.4}{2} \Delta t + \frac{20.4 + 19.4}{2} \Delta t \\ & = \frac{1}{2} \Delta (24.9 + 23.7 + 22.6 + 21.5 + 20.4 + 19.4) \\ & = 110.35 \text{ billion barrels} \end{aligned}$$

5. [10 points] Use the NUMINTG program on your calculator and the idea of limit of sums to calculate this same integral precisely, to two decimal places, using the trapezoidal method. How many trapezoids did you need to achieve this accuracy?

Fewer than 40 trapezoids are needed to achieve two-decimal accuracy:

number of trapezoids:	10	20	40
estimate:	110.26	110.25	110.25

6. [10 points] Finally, compute this integral again, this time by finding an antiderivative of $r(t)$.

$$\int_5^{10} 32e^{-0.05t} dt = \frac{32}{\ln e^{-0.05}} e^{-0.05t} \Big|_5^{10} = \frac{32}{-0.05} (e^{-0.05 \cdot 10} - e^{-0.05 \cdot 5}) = 110.25$$

7. Compute these antiderivatives:

(a) [10 points] $\int \left(1 + \frac{1}{x} + \frac{1}{x^2} \right) dx$

$= x + \ln x - x^{-1} + c$ (the third term in the integral is equivalent to x^{-2})

(b) [10 points] $\int (4.63x^2 - 9.2x + 6) dx$

$= 1.54x^3 - 4.6x^2 + 6x + c$

Form B

1. (a) [10 points] Use the data in the table below to create a cubic model for $N(t)$ = yearly emissions (in metric tons) of nitrogen oxides in the United States in year t since 1940. Use two significant digits for each of the coefficients.

Year	1940	1950	1960	1970	1980	1990
Emissions	6.9	9.4	13.0	18.5	20.9	19.6

The calculator gives $N(t) = -0.00036t^3 + 0.023t^2 + 0.013t + 7.0$ for the model.

- (b) [10 points] It is clear from the table that the emissions levels were increasing until recently, when they began to decrease. Compute the formula for the first derivative of $N(t)$ and use it to determine the year in which emissions of nitrogen oxides reached a maximum and what that maximum emission level was.

We find $N'(t) = -0.00108t^2 + 0.046t + 0.013$; setting this equal to zero gives a quadratic equation whose solutions are

$$t = \frac{-0.046 \pm \sqrt{0.046^2 - 4(-0.00108)(0.013)}}{2(-0.00108)} = -0.28, 42.87$$

The second solution is the only one in the domain of interest, and gives the value of the maximum. In 1982, the maximum emission level was $N(42.87) = 21.46$ metric tons

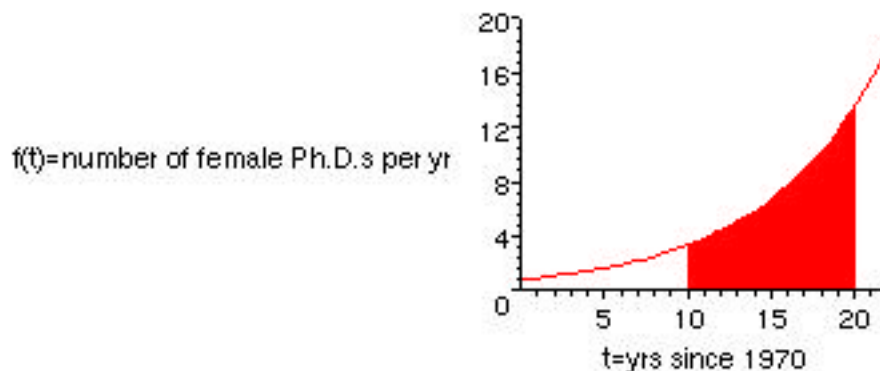
- (c) [10 points] Compute the formula for the second derivative of $N(t)$ and use it to determine the year in which the rate of increase in emissions was greatest and what that greatest rate of increase was.

We find $N''(t) = -0.00216t + 0.046$; setting this equal to zero leads to $t = 21.30$. That is, the rate of growth in emissions was greatest in 1961, when it was growing at the rate of $N'(21.30) = 0.50$ metric tons per year.

The table below gives partial figures for the number of female Ph.D.s granted in computer science over the last 30 years or so.

Year	1970	1973	1976	1979	1982	1985	1988	1991
Ph.D.s/yr	1	1	2	3	4	7	10	17

2. [10 points] Fit an exponential model to this data for $f(t)$ = number of female Ph.D.s granted per year for year t since 1970 and write the equation below; use three significant digits for the coefficients. (Make 1970 the baseline year.) Then sketch a plot of this model curve, labeling axes appropriately. Shade the region between the horizontal axis and the curve over the interval $10 \leq t \leq 20$.



3. [10 points] Write the correct integral notation to represent the shaded area in your graph in #2. Then write a sentence describing what this quantity measures.

$$\int_{10}^{20} 0.825(1.15^t) dt = \text{total number of female Ph.D.s granted between 1980 and 1990}$$

4. [10 points] Tabulate values of $f(t)$ for $t = 10, 12, 14, 16, 18, 20$ (to one decimal place accuracy). Use these numbers to give an estimate for the value of the integral in #3 by means of 5 trapezoids. Show your arithmetic.

t	10	12	14	16	18	20
$f(t)$	3.3	4.4	5.8	7.7	10.2	13.5

$$\begin{aligned} \int_{10}^{20} 0.825(1.15^t) dt &\approx \frac{3.3+4.4}{2} \Delta t + \frac{4.4+5.8}{2} \Delta t + \frac{5.8+7.7}{2} \Delta t + \frac{7.7+10.2}{2} \Delta t + \frac{10.2+13.5}{2} \Delta t \\ &= \left(\frac{1}{2} \Delta t (3.3 + 4.4 + 5.8 + 7.7 + 10.2 + 13.5) \right) \Delta t \\ &= 73 \text{ Ph.D.s between 1980 and 1990} \end{aligned}$$

5. [10 points] Use the NUMINTG program on your calculator and the idea of limit of sums to calculate this same integral precisely, to two decimal places, using the trapezoidal method. How many trapezoids did you need to achieve this accuracy?

Fewer than 100 trapezoids are required to achieve two-decimal accuracy:

number of trapezoids	10	20	40	80	100
estimate	72.85	72.76	72.74	72.73	72.73

6. [10 points] Finally, compute this integral again, this time by finding an antiderivative of $f(t)$.

$$\int_{10}^{20} 0.825(1.15^t) dt = \frac{0.825}{\ln 1.15} (1.15^t) \Big|_{10}^{20} = \frac{0.825}{\ln 1.15} (1.15^{20} - 1.15^{10}) = 72.73$$

7. Compute these antiderivatives:

(a) [10 points] $\int \left(\frac{9.245}{x} - 1.472 \right) dx$

$$= 9.245 \ln x - 1.472x + c$$

(b) [10 points] $\int (4.63x^3 - 9.2x^2 + 6) dx$

$$= 1.1575x^4 - 3.07x^3 + 6x + c$$