1. (11 points) The table shows the number of packages delivered by a shipping company each week after a strike by delivery drivers.

<table>
<thead>
<tr>
<th>Week</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Packages (thousands)</td>
<td>180</td>
<td>130</td>
<td>95</td>
<td>70</td>
<td>60</td>
<td>62</td>
<td>75</td>
</tr>
</tbody>
</table>

a. Examine the data set. What would be the best model? Carefully explain, making sure you tell me why you ruled out the other models.

**Sol:** This is quadratic. This is always concave up so it can’t be linear (since linear models have no concavity) and can’t be cubic or logistic, both of which have changes in concavity. This leaves exponential, quadratic, or logarithmic. Exponential models level off to 0 in one direction and this doesn’t have that behavior. Logarithmic models go to positive infinity in one direction and negative infinity in the other, which this graph isn’t doing (also, log models aren’t defined at 0). This leaves only quadratic.

b. Calculate the best fit model and write it here (recall: 4 decimal places!) Make sure to define the variables you use, and include the units of measure.

**Sol:** Let \( x = \text{weeks}, f(x) = \text{number of packages delivered in 1000's}. \) Then 

\[
 f(x) = 6.3095x^2 - 55.2143x + 179.6190. 
\]

2. (11 points) The mouse population in the subways of a certain city is currently estimated at 5300. Rodent control experts estimate that the population grows by 4% each month. Find an exponential model, denoted \( m(x) \), for the number of mice living in the subways, carefully explaining how you found all of your variables.

**Sol:** \( m(x) = ab^x \). Now we know that \( (b - 1) \cdot 100\% = 4\% \implies (b - 1) = \frac{4\%}{100\%} = 0.04 \implies b = 1.04 \). Thus we know \( m(x) = a(1.04)^x \). Now at time 0 there are 5300 mice. Thus \( m(0) = 5300 \). But from the model, \( m(0) = a(1.04)^0 = a \cdot 1 = a \). Thus \( a = 5300 \) and we have our model: \( m(x) = 5300(1.04)^x \).

4. (18 points) The table shows the number of bank failures, \( f(x) \), in the United States from 1988 through 1995 (note: 1992 is missing!), where \( x \) is years since 1985.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Failures</td>
<td>221</td>
<td>207</td>
<td>169</td>
<td>127</td>
<td>41</td>
<td>13</td>
<td>6</td>
</tr>
</tbody>
</table>

a. Examine a scatter plot of the data (Input your data with \( x \) being years since 1985). Is the plot increasing or decreasing? Discuss the concavity of the graph (being specific). Explain why a logistic model is appropriate for these data.

**Sol:** The plot is decreasing. In starts out concave down and becomes concave up. Since it has a change in concavity it could be either cubic or logistic. But it seems to level off, so logistic is appropriate.

b. Find a logistic model to fit the data and write it here. How well does the model fit?

**Sol:** \( y = \frac{234.7788}{1 + 0.0042e^{8.892x}} \). This fits very well.

c. In approximately what year does the inflection point appear?

**Sol:** About \( x = 6 \) so 1991.

d. Find the following using your model from part b:

- \( \lim_{x \to -\infty} f(x) = 0 \)
- \( \lim_{x \to +\infty} f(x) = 234.7788 \)
5. (16 points) The average mobile-phone local monthly bill in the U.S. between 1995 and 1998 is modeled by $B(x) = -3.963x + 51.172$ dollars, where $x$ is the number of years after 1995.

a. Find and interpret the vertical axis intercept. Explain. (about one sentence i.e. how did you find your answer – saying you did this on your calculator is not valid here).

**Sol:** $B(0)=51.172$. In 1995 the average mobile-phone local monthly bill in the U.S. was $51.172$. 