Directions:

- You have 50 minutes in which to complete this exam.
- Make sure that you show all work, or risk losing credit.
- Be sure to answer all questions asked.
- **No calculators allowed on this exam.**
- Give all units of measurement, whenever appropriate!

Good luck!
1. (10 points) A college student takes a summer job in a local factory as a toy assembler. The student does piecework, i.e. is paid per completed piece. Let $P$ be the number of pieces the student has assembled $x$ hours from the start of the student's shift. Let $T$ be the student's earnings for the day when the student has completed $P$ pieces. Four hours after the shift starts, the student has assembled 25 pieces but that number was decreasing by 2 pieces per hour. The rate of change of the student's earnings is a constant $0.20$ increase per completed piece. Identify the following values at four hours after the start of the shift. **Show all work** as needed and **give all units**. Note: part of this problem is a (first form) chain rule problem.

a. \[
\frac{dP}{dx} =
\]

b. \[
\frac{dT}{dP} =
\]

c. \[
\frac{dT}{dx} =
\]

d. Write a sentence interpreting your result from part (c) in plain English.
2. (20 points) Use the Four-Step method to find the derivative of \( f(x) = -6x^2 + 18 \). Label all four steps with a short description (like we did in class). Do not skip steps and make sure to SHOW ALL WORK! Notation counts in this problem!
3. (25 points) We want to draw the slope graph of the following function, \( f(x) \):

a. At which \( x \) values are the tangent lines to the curve horizontal? What does this tell us about the slopes?

b. Where is the graph decreasing? What does this tell us about the slopes?

c. Where is the graph increasing? What does this tell us about the slopes?

d. Where (if anywhere) are the \( x \)-value(s) of local (relative) maxima?

e. Where (if anywhere) are the \( x \)-value(s) of local (relative) minima?

f. Draw the number line indicating where the slopes are zero, dne, positive and negative.

g. At which \( x \)-values do we have an inflection point?
   - For each inflection point, is it steepest or least steep place (in the surrounding area)?
   - For each inflection point, does it translate to a maximum, minimum or neither on the SLOPE GRAPH.

h. Using the information in parts a-g above, sketch the slope graph of the function \( f(x) \). Make sure that you have included all important points that need to be on the slope graph.
4. (45 points) Find the derivative formula for each function, using the derivative rules (i.e. not the four-step method.) Make sure that you include the correct derivative notation for each answer! You do not need to simplify after you have completed taking the derivative. Note: this problem is continued on two pages.

a. \[ y = 200x^4 - 40x^4 + 81 + \frac{52}{x^2} \]

b. \[ L(t) = 0.1t^{-6.5} + \frac{8}{t^{10}} - \frac{\sqrt[5]{t^3}}{6} + \pi + 57t + \frac{7}{3\sqrt[5]{t}} \]

c. \[ P(t) = 112 \ln t + e^{12} + 14e' - 5 \left( \frac{12}{5} \right)' + \pi^e \]
4. continued

d. \( q(m) = \ln(m^{10} + m - 13) \)

e. \( a(b) = 10^6 \cdot (15b - 13) \)

f. \( g(x) = (5x + 20)^5 (6x^2 + 3)^{11} \)