MATH 150-05
Exam IV

Name: ___________________________

Directions:

- You have 50 minutes in which to complete this exam.
- Make sure that you show all work, or risk losing credit.
- Be sure to answer all questions asked.
- Give all units of measurement, whenever appropriate!

Recall the quadratic formula: if we want to solve $ax^2 + bx + c = 0$, the roots of this equation are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Good luck!
1. (26 points) Let \( f(x) = \frac{3}{5}x^5 + \frac{5}{4}x^4 - \frac{28}{3}x^3 - 20 \). We want to find all relative and absolute extrema over the interval from -5 to 3, using CALCULUS.

a. Define: one obtains a critical point on a function \( f(x) \) wherever ____________________________

b. Find \( f'(x) = \)

c. Find the \( x \)-coordinate(s) for all critical points on the graph of \( f(x) \) using calculus and showing all work. (I.e. you must show all work and use calculus and not your calculator to find the exact \( x \)-values of the solutions.) Be careful and make sure that you use the definition of critical point from part a above.

d. The following is the graph of our function, \( f(x) = \frac{3}{5}x^5 + \frac{5}{4}x^4 - \frac{28}{3}x^3 - 20 \) over the interval from -5 to 3.

- Find all relative extrema on this graph by utilizing previous results from this problem. For each relative extreme point that you find make sure you tell me if it is a relative maximum or if it a relative minimum. Also, your answer should look like: we have a relative maximum at of _____. Your answers should be the actual answers – not approximations!

- Find all absolute extrema on this graph by utilizing previous results from this problem. Follow the same instructions for answers as were used for relative extrema above.

e. Suppose that a certain function \( h(x) \) is such its the first derivative is \( h'(x) = \frac{x}{x - 6} \). Find all critical points, using calculus. Note: I haven’t told you what the original function is – just what its first derivative is, so you can’t figure out anything more from this than just the critical points.
2. (17 points) a. Let \( f(x) = -\frac{x^7}{6} + \frac{x^6}{5} - 0.01 \). We want to find the exact values of the \( x \)-coordinates of the inflection points of \( f(x) \).

- Find \( f'(x) = \)

- Find \( f''(x) = \)

- Find the \( x \)-coordinate(s) for all possible inflection points on the graph of \( f(x) \) using calculus and showing all work. (I.e. you must show all work and use calculus and not your calculator to find the exact \( x \)-values of the solutions.) Be careful and make sure that you use the definition of (possible) inflection point from part d above.

- Here is the graph of \( f(x) = -\frac{x^7}{6} + \frac{x^6}{5} - 0.01 \).

For each of the possible inflection points that you obtained above, tell me if they are actual inflection points, and explain how you know.

b. Suppose that a certain function \( g(x) \) is such that \( g''(x) > 0 \) for all values of \( x \). What does this mean about the look of \( g(x) \)? (I.e. what does the fact that \( g''(x) > 0 \) mean?)
3. (25 points) The rate of change of the temperature during a thunderstorm, for the two hours after the storm began is given by \( T(h) = -3h^4 + 5h^3 + 10.35 \) °F per hour where \( h \) is the number of hours since the storm began.

a. The following is a graph of \( T(h) \):

- What are the units on the area under \( T(h) \) but above the x-axis from \( h=0 \) to 2? Show all work.

- What does the area of the region under \( T(h) \) but above the x-axis from \( h=0 \) to 2 represent? (One sentence – be careful in your phrasing!)

b. We want to use 5 midpoint rectangles to approximate the area below \( T(h) \) but above the x-axis from \( h=0 \) to 2.

- First, what is \( n \) here? ______

- Next, what is \( \Delta x \) here (showing all work)?

- On the graph above, carefully draw the five midpoint rectangles (points are given for accuracy).
- Fill in the following table:

<table>
<thead>
<tr>
<th>Rectangle #</th>
<th>Midpoint</th>
<th>Height of rectangle</th>
<th>Width of rectangle</th>
<th>Area of rectangle</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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- Find the total area of the rectangles, giving units on your answers.

c. How could we get a better estimate than the one we just found in part b? (Don’t do it – just explain what to do.)
4. (12 points) The following is a graph of some function $f(x)$ on the interval from 0 to 12.

Area A = 0.95
Area B = 10.20
Area C = 20.35
Area D = 25.55

Note that:
- Area A goes from 0 to 2
- Area B goes from 2 to 6
- Area C goes from 6 to 10
- Area D goes from 10 to 13

Find the following, showing work as necessary:

a. $\int_{6}^{10} f(x) \, dx =$

b. $\int_{2}^{6} f(x) \, dx =$

c. $\int_{10}^{13} f(x) \, dx =$

d. $\int_{6}^{13} f(x) \, dx =$

e. $\int_{2}^{10} f(x) \, dx =$

f. $\int_{0}^{13} f(x) \, dx =$
5. (20 points) Find the following antiderivatives, showing work as needed:

a. \( \int (6x^9 - 3x) \, dx = \)

b. \( \int \frac{8}{x} \, dx = \)

c. \( \int (e^{2x} + e^x + e) \, dx = \)

d. \( \int \left( \frac{13}{\sqrt{u}} + \sqrt[3]{u} + 13 \right) \, du = \)

e. \( \int \left( 8e^t + 3 \cdot \pi^t \right) \, dt = \)