

Mathematical Perspectives:
Mathematics and the Creative Imagination
MATH 125-02

Fall 2011

Syllabus

Time: 12:30-1:20 MWF

Classroom: Alter 321

Instructor: Dr. Dena Morton

Office: Hinkle 108

Phone: x3674 (Note: I do not check my voice mail very often.)

Office Hours: By **appointment** (including phone appointments) or **without an appointment on**

- Monday 10:30-11:20, Wednesday 10:30-11:20, 1:30-2:30, Friday 10:30-11:20, 1:30-2:00

e-mail: morton@xavier.edu

Note: this is the best way to reach me – I check my e-mail on a regular basis.

Web Page: <http://cerebro.xu.edu/~morton/a/classes.html>

Note: I update my webpage every day – **all homework assignments and readings** are always posted online. Also, check out the beautiful mathematical pictures!

Purpose and Content: The general objective of Mathematical Perspectives is to provide students with experiences characteristic of the mathematical enterprise, and to do so at a depth that allows successful students to appreciate the aesthetic beauty of mathematical truth as well as its timeless and user-independent nature. In Mathematics and the Creative Imagination we will be exploring creativity in Mathematics. The Heritage Illustrated Dictionary defines creativity as "Characterized by originality and expressiveness, imaginative". Too many students associate mathematics with algorithmic thinking and as a pursuit that is abstract for the sake of abstraction and thus devoid of any real-world meaning. But they have never considered many of the topics of Mathematics that makes its study so enriching. We will learn that Mathematics is more than just algebraic manipulation, but rather that it is marked by originality, imagination, beauty, and the unexpected!

We will explore many topics in mathematics that will be new to most students:

- Game Theory: mathematics is present in many unexpected places, including in games like NIM.
- The Four Coloring Problem: an unexpected place in which to find mathematics – in maps!
- Patterns and Figurate Numbers: Useful patterns are all around us. What would happen if we wanted to add all the numbers from 1 to 1,000,000? Is there a quick and easy way to do this? How can we represent this as a geometric figure?
- Topology: we will construct Mobius strips (of many different kinds) and discuss homeomorphisms (as they relate to basic shapes). We will learn that topology arose as an offshoot of geometry: What would happen if we stopped worrying about distance and area and allowed shapes to be elastic?
- Number Theory: we will discuss the classical proofs that square root of two is irrational and that there are infinitely many primes. They will encounter these proofs again in "The Mathematician's Apology" by G.H. Hardy. We will discuss the creative process utilized by Ramanujan. We will discuss some classical number theory: the sieve of Eratosthenes, prime numbers, perfect numbers, etc. We will read excerpts from "The Man Who Loved Only Numbers" by P. Hoffman. By discussing the language used by Erdos (e.g. "died" means "stopped doing mathematics") students will be able to gain more insight into the creative mind of a mathematician.
- Dimensionality: we will read "Flatland" by E. Abbott and discuss how we can imagine four-dimensional space. We will examine how we could use the geometry of the familiar Euclidean world to explore a geometry of the mind.
- Cryptography: Using the number theory already developed we will explore some classical cryptosystems (Caesar cipher etc.) We will read excerpts from "The Code Book" by Simon Singh in order to trace the history of cryptography. Next we will examine the ways that cryptographers create and break newer and better codes.
- Group Theory: we will read the article, "Group Theory and the Postulational Method" by Denbow and Goedicke in order to learn about the non-abelian dihedral groups. This will illustrate how real phenomena can display unfamiliar arithmetic properties. We will utilize cutouts of triangles, squares, rectangles, etc in order to represent dihedral groups. Last, we will examine the algebraic construct of "loops" (and see how it relates to the picture on the front of these materials and on my webpage).

- Logic: First, we will use logic to solve a murder mystery. Then we will read "Alice in Wonderland" by Lewis Carroll and use the language of predicate logic to construct the "logic" of Wonderland. We will discuss more logic (and, or, implies, tautologies, contradictions, etc.) as it applies to thinking processes.
- Non-Euclidean Geometry (if time allows): We will discuss spherical and hyperbolic geometry in order to understand how new areas of mathematics are created/discovered.
- Fractals (if time allows): We will discuss how beautiful mathematical fractals are created.

By the historical discussions of how each topic of mathematics arose and a thorough discussion of the creativity involved, each of these topics has been selected in order to facilitate students' understanding of how one actually DOES mathematics, how much creativity is involved, and how imaginative, beautiful, and interesting it truly is.

Texts: *Alice's Adventures in Wonderland and Through the Looking Glass*, by Lewis Carroll (any edition)
A Mathematician's Apology, by G. H. Hardy (with foreword by C.P.Snow – Cambridge University Press Jan, 1992) [ISBN-10: 0521427061](#)
The Man Who Loved Only Numbers by P. Hoffman [ISBN-10: 0786884061](#)
The Code Book: The Science of Secrecy from Ancient Egypt to Quantum Cryptography by Simon Singh (Anchor Aug, 2000) [ISBN-10: 0385495323](#)
Flatland by E. Abbott (any edition)
Group Theory and the Postulational Method by Denbow and Goedicke (over e-reserves)
 The codeword to access this article is "creativity".
 Course Materials packet: **must be brought to class each meeting**

Assignments: During the course of the semester, the student will read the course texts and the assigned article (the article is already online). On the date due, we will have an in-depth discussion of the article or book. Students will write critiques for some of the articles (I will let you know which!). In these cases, the critique should be handed in (at the beginning of class) at the time of discussion of the article. Critiques should be **neatly typed, stapled, and checked for composition, content, spelling and grammar.** **Critiques that are misspelled or grammatically incorrect will lose points. Formal critiques which are unreadable due to poor proofreading will be assigned a failing grade.** The critiques should contain the reader's general impression of the article, a list of points not fully understood, a list of topics the reader might want further developed, and any questions that might arise. Additionally, each critique must include a discussion of (at least) one new topic (i.e. a topic learned from the reading) that you found to be interesting/exciting/totally cool. **The official criteria for critiques are given at the end of this course materials packet.** For those readings for which no formal critique is assigned, other instructions will be given. Late papers and critiques may not be graded unless you have an acceptable excuse, which must be submitted to me BEFORE the paper is due. Even if I do accept your late paper, I might drop your grade depending on the lateness and the circumstances.

One does not learn or attain proficiency in mathematics at any level unless one does problems. Therefore, problems will frequently be assigned. Homework assignments are not usually collected, but I do reserve the right to collect them and grade them. Detailed solutions will be handed out to most assignments, and the answers to many are at the end of this packet.

Class Activities: Classes will consist of group activities, discussion, individual activities, and lectures.

Quizzes: Weekly quizzes will be given on Fridays. The lowest quiz score will be dropped, so makeup quizzes will not be given. Quizzes will not be given during exam weeks. Some quiz problems will involve statements of definitions, since if you don't know the definitions, you cannot possibly expect to do the mathematics.

Exams: There will be three exams given throughout the semester, each consuming an entire class period. There will also be a comprehensive final exam. If you must miss an exam for religious or academic reasons, or in cases of illness or emergency, you must submit a written excuse. A makeup may be scheduled -- this will be decided on a case-by-case basis.

Grading: Quiz performance constitutes 10% of your final score. Each exam will be worth 14% of your final grade. Papers, class participation, and homework constitute 24% of your final score. The final exam is worth 24% of your grade.

Each exam will be curved separately and assigned a number grade between 0.0 (the lowest possible F) and 5.0 (the highest possible A). I will announce the cutoffs when returning the exam. If, for example, the cutoff for an A is 87 and the cutoff for a B is 71 and you get an 83, then the number grade corresponding to your 83 would be a 3.75 (B corresponds to 3.0 and you are $12/16 = .75$ of the way to the next cutoff). The homework and quizzes will be treated similarly. The total course grade may be curved further (that is, a 3.9 might result in an A or A- in the course), but the resulting curve will never lower your grade (that is, a 4.1 would always result in at least an A- in the course). I reserve the right to assign a grade of "F" to any student who earns less than 50% on the final exam.

Some assignments will be assigned grades using the following scale: excellent / exceptional, very good, good, ok, passing, not acceptable. Excellent/exceptional usually counts as A+, very good as A-, good as B, OK/satisfactory as C, minimal passing as D, failure as F.

Important Dates (Exams are Tentatively Scheduled):

Wednesday, Aug. 24	First day of class
Monday, Sept. 5	Labor Day Holiday (no classes)
Wednesday, Sept. 21	Exam I
Friday, Oct. 14	Fall Holiday (no classes)
Friday, Oct. 21	Class cancelled
Friday, Oct. 28	Exam II
Monday, Nov. 21	Last day to withdraw
Wednesday, Nov. 23-Friday, Nov. 25	Thanksgiving Holiday (no classes)
Friday, Dec. 2	Exam III
Friday, Dec. 9	Last day of classes
Monday, Dec. 12	Study Day
1:00-2:50 Wednesday Dec. 14	Final Exam

Attendance: Class attendance is crucial. Lectures include the introduction and explanation of new topics, explorations of proofs, and solutions of discrete mathematics problems. Class notes are to be used in conjunction with the text, in order to elicit a fuller understanding of discrete mathematics.

Please be courteous and come to class on time!

University policies on attendance are stated in section 5 of the undergraduate handbook (available online).

Missed Classes: If you must miss a class due to illness or an emergency, you must first get a copy of the notes from one of your classmates. (If you do not know anyone in the class, I will help you contact someone to get notes.) Review the missed notes, and write detailed questions as you are reading them. I will be happy to answer all of your questions (as many as you would like to ask!), but I cannot re-lecture for you. As noted above, quizzes cannot be made up.

Group Work: Working in a group can be beneficial for everyone involved, provided that you do not abuse the privilege. Make sure that everyone in your group is making a contribution. Do not copy answers from one another, as this will only backfire against you come test-time (and is also cheating)! Instead, let concepts gel after group discussion, and then write up the solutions by yourself.

Academic Honesty: You are expected to conduct yourself with integrity in this course. Cheating will be dealt with as harshly as University regulations permit; measures will be taken during exams to prevent cheating. Students are directed to the undergraduate handbook for further information. Note: talking during an exam (to anyone other than me) is grounds for a failing grade on the exam. **Using (uncited) web-pages to write your critique is cheating and plagiarizing!** The best critiques are written from your own experience.

Calculators: You will need some sort of calculator for the segments on number theory and cryptography – a TI-83/84 is suggested. **Cell phone calculators are not allowed.** You may not have **any** programs on your calculator. If you have old programs, you must transfer them to a disc and reset all calculator memories at the beginning of exams.

Cell phones: **Please turn all cell phones off during class (no texting either, please).** Cell phones must be away during exams.

How to Do Well in this Course: Come to class! Come visit me during office hours! Read the books and articles! Try the problems! Smile! Study hard! Read your class notes! Make sure you keep up with the material in class! Review your class notes! Don't Panic! *Enjoy!* Most important of all, if you feel that you are falling behind, or that you do not understand a certain topic, or if you would just like to discuss a mathematical idea (or anything else), come to visit me in my office. That's why I am here!☺